

‘CHESS STUDIES’ FOR STRING QUARTET:

Composition based on chess

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Resumo: Este artigo propõe uma primeira abordagem sobre as possibilidades de adaptação de elementos matemáticos derivados do xadrez para a composição musical a partir da análise da obra *Chess Studies* for String Quartet, escrita em 2020 pelo 1º autor deste artigo. A estrutura dos seus quatro movimentos surge da sobreposição da geometria derivada de uma sequência de movimentos para a execução do xeque-mate do Bispo e do Cavalo num quadrado mágico do tabuleiro 8x8, neste caso, o quadrado mágico de Al-Zarquali (século XI). A análise aprofunda a evolução do material musical utilizado, mostrando a tendência para dar origem a simetrias de alturas e fragmentos temporais entre os diferentes movimentos da composição. Estas simetrias dão origem a analogias com o conceito de *quiralidade*, característico da geometria do xadrez. Estruturas que apresentam quiralidade surgem na obra tanto no espaço sonoro como no espaço temporal.

Palavras-chave: Composição Musical, Xadrez, Quadrado Mágico, Geometria, Quiralidade.

Abstract: This article proposes a first approach to the possibilities of adapting mathematical elements derived from chess to musical composition based on the analysis of the composition process of the work *Chess Studies* for String Quartet, composed in 2020 by the 1st author of this paper. The structure of its four movements arises from the superimposition of the geometry derived from a sequence of moves for the execution of Bishop and Knight checkmate on a magic square of the 8x8 board, a magic square of Al-Zarquali (11th century). The analysis delves into the evolution of the musical material used, showing the tendency to give rise to symmetries of pitches and temporal fragments between the different movements of the composition. These symmetries give rise to analogies with the concept of *chirality*, characteristic of the geometry of chess. Structures that show chirality appear in the work both in sound and temporal space.

Keywords: Music Composition, Chess, Magic Square, Geometry, Chirality.

Since the birth of the dodecaphonic technique, and especially with the development of composition in the 1950s, the use of mathematical processes in composition has had an important role in avant-garde music. Different serialist and post-serialist tendencies, as well as minimalist, spectral, and new complexity music, among others, have produced works based in one way or another on the artistic adaptation of mathematical structures to composition, the example of Iannis Xenakis (1922-2001) being perhaps the most paradigmatic one:

It is not so much the inevitable use of mathematics that characterizes the attitude of these experiments, as the overriding need to consider sound and music as a vast potential reservoir in which a knowledge of the laws of thought and the structured creations of thought may find a completely new medium of materialization, i.e., of communication (Xenakis, 1992, Preface ix).

On the other hand, the mathematical component of music has been an important object of study since the origins of composition. Throughout history, the deepening into the mathematical nature of music has sometimes produced works with a compositional trend to consider music as an abstract phenomenon of a mathematical nature. In the 20th century, kinds of new composition techniques have made the intrinsic mathematics of the musical phenomenon more evident in composition. Works created with these new techniques, exhibit a character of intellectual representation of the sonic field that transcends the nineteenth-century aesthetic ideal, entering the field of the search for knowledge of natural laws and their manifestation in the art of music, as Webern expressed in *The Path to the new Music*:

And the works that endure and will endure for ever, the great masterpieces, cannot have come into being as humanity, more's the pity, imagines. [...] To put it more plainly, man is only the vessel into which is poured what "nature in general" wants to express. You see -I would put it something like this: just as a researcher into nature strives to discover the rules of order that are the basis of nature, we must strive to discover the laws according to which nature, in its particular form "man", is productive. And this leads us to the view that the things treated by art in general, with which art has to do, are not "aesthetic", but that is a matter of natural laws, that all discussion of music can only take place along these lines (Webern, 1960, p. 11).

The game of chess originally constituted a symbolic representation of war, where a certain geometry rules the movement of the different figures on the board. From the pattern of the moves of

the pieces countless geometric structures can be produced, those related to symmetry, particularly with the concept of “chirality”, intrinsic to chess, are of great interest. The other basic element of the game, the 64-square board, can be represented as a square matrix of 8x8 squares. And particularly can be constructed as a “magic square”¹ of order 8 whose magic sum is 260. The interrelationship between the geometries of the moves of the pieces and the arithmetic feature of the magic square can constitute a mathematical basis on which it is possible to approach the treatment of the variables of the composition. The number of possibilities of geometry in chess is enormous², which represents a wide range from which specific aspects of the composition process of a work in its entirety can be addressed. In the work studied in this article, a first approach to the possibilities of adapting chess material to composition has been made.

The work *Chess Studies* for String Quartet, written in 2020 by the 1st author of this paper, consists of four pieces, independently viewed as musical adaptations of four chess studies. In their composition, the same sequence of piece moves from each of the four sides of the chessboard square was used (see FIGURE 2). In every study, the types of musical materials are the same and are arranged in the same way. Although they are different internally, the four studies have the same length of 260 quarter notes, a quantity deducted from the magic sum of the square.

¹ A square matrix of numbers is a “magic square” if the sum of the numbers in each row, each column, and main diagonals are equal. The “magic sum” depends on the number of rows and columns of the matrix and is obtained under the formula $n \cdot (n^2 + 1) / 2$, where n represents the number of rows and columns. For example, in a square of order 8 (8 rows and 8 columns), the magic sum is 260.

² An estimation of the number of possible positions in chess is represented by the Shannon number: 10^{120} (Shannon, 1950).

1. Application of chess material

The material used as a basis for *Chess Studies* has been extracted from a sequence of moves for the Bishop and Knight checkmate, which is the most difficult basic checkmate to deliver³. The sequence has been extracted from a computer-generated database of *Endgame Tables*, which from a chess point of view represents a perfect sequence of moves⁴. The chosen sequence of moves shows in its geometry a clear coordination of the pieces to deliver the checkmate, which shows an aesthetic beauty in the overall geometry resulting from their combination. On the other hand, the moves of the losing side have not been considered because they, forced to delay the inevitable checkmate, show a somewhat chaotic geometry. The sequences of moves of the Knight, Bishop and King, from which the composition has been structured are the following⁵:

Knight: Nc1-c3, Nc3-e4, Ne4-d6, Nd6-f7, Nf7-e5, Ne5-d7, Nd7-b6, Nb6-a4, Na4-c5, Nc5-a6
Bishop: Bf1-c4, Bc4-d3, Bd3-h7, Bh7-d3, Bd3-e2, Be2-b5, Bb5-a4, Ba4-d7, Bd7-c6
King: Ke1-e2, Ke2-e3, Ke3-f4, Kf4-f5, Kf5-f6, Kf6-e6, Kde-d6, Kd6-c6, Kc6-c7, Kc7-b6

1.1. Numeric matrix of the magic square

A magic square from the 11th-century⁶ Andalusian astronomer Al-Zarqali (Comes & Comes, 2009) has been chosen to construct the numerical matrix in which the moves of the pieces are executed. As has been said, the sum of its horizontal ranks and vertical files, as well as its two main

³ The basic checkmates are those possible checkmates (not all combinations of pieces can checkmate) that take place when the winning side has one or two pieces in addition to the King against the lone King of the opposite side.

⁴ The *Endgame Tablebases* are databases of positions of up to 7 pieces (Syzygy: <https://syzygy-tables.info>) in which all the possible moves have been analysed by computer. In this way, it has been possible to deduce the optimal moves within a given position to determine if one side is a winner or if the position is a draw. These Endgame Tablebases represent, to date, the only positions the machines have managed to decipher.

⁵ In chess, the annotation of the moves is made by the so-called "algebraic system". Ordering the horizontal from left to right with the letters "a" to "h" and the vertical from bottom to top with the numbers from 1 to 8, it is possible to write down the moves indicating the square of origin and arrival of the piece. It is important to note that in chess horizontal rows are called "ranks" (rank 1 2, 3, etc.) and vertical columns are called "files" (file a, b, c, etc.).

⁶ The reference to the construction of magic squares from the moves of the chess pieces dates to the time of the Persian and Arab rules of the game (perhaps even to its possible origins in India), which were modified in the Renaissance in Western Europe, leading to the configuration of the modern moves of the pieces.

diagonals, is 260. Looking at the square from each of its four perspectives (see FIGURE 2), four different number sequences can be seen on the bottom rank. Their numbers are used as values to determine the duration of the sections in each study. Those durations are counted in quarter notes with a general metronomic indication of quarter=60. The differences between the numerical series mean that although the chess moves are identical in the different studies, each shows differences in the development of its temporal structure even though the same musical material is assigned to each section in all the studies. The numerical sequences from the first rank of the magic square, starting from each side, are as follows:

Study n. 1:	64	2	3	61	60	6	7	57
Study n. 2:	57	16	24	33	25	48	56	1
Study n. 3:	1	63	62	4	5	59	58	8
Study n. 4:	8	49	41	32	40	17	9	64

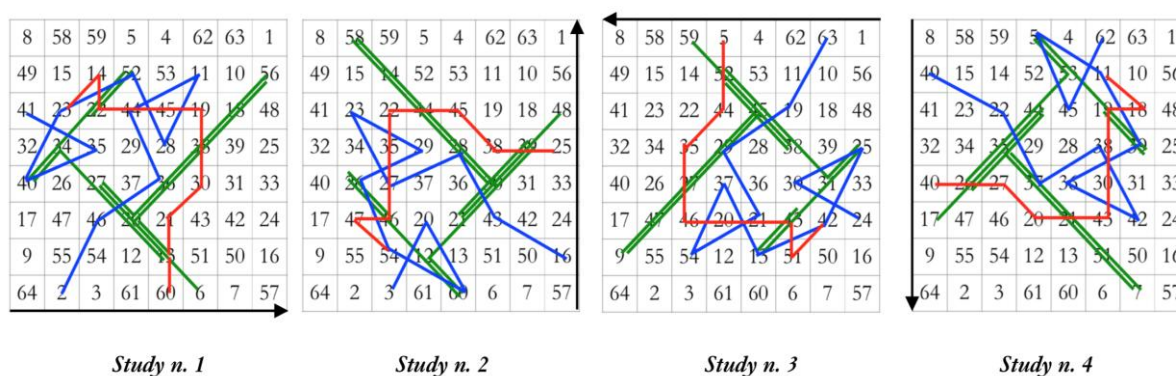
Each study consists of eight sections whose duration in quarter notes is expressed by the sequence of numbers in the bottom rank: in the first study, following the sequence 64 - 2 - 3 - 61 - 60 - 6 - 7 - 57. Following the moves of the figures, according to the numbers represented in the squares through which they move, both the duration and the change of the pitches of the corresponding events are determined. For the development of the pitches throughout the piece, the moves of the Knight, Bishop and King have overlapped on the magic square. According to the numbers represented in the squares they reach the pitches have been assigned in ascending order to the numbers 1 through 64, starting from the lowest pitch of the Violoncello (C₂), as shown in FIGURE 1.

FIGURE 1 - (A. Hortigüela), *Chess Studies*. Range of pitches.



Each musical event is assigned a change between the pitch indicated by the number of the initial square (reading the magic square from left to right from each perspective) and the number of the square which reaches. The duration of the event is defined by the number represented in the bottom rank. There are events whose duration is restricted to one section and others that last two or more sections. For example, at the beginning, the Bishop moves from 40 to 52 extending to the third file. This means that the material corresponding to its move extends beyond the first section coexisting with the materials of the second and third sections. This phenomenon of maintaining a material in the following section is frequently manifested in the work. The structure resulting from the superposition of the moves of the pieces on the magic square is represented in FIGURE 2⁷:

FIGURE 2 – (A. Hortigüela), *Chess Studies*. Superposition of the sequence of moves on the magic square.



At the beginning of the first study, five different moves of the pieces correspond to as many musical events (see FIGURE 2). One of them is a Knight move that goes from square 41 to 35. According to the bottom rank, the duration of the event is $64+2=66$ quarters, between pitches 41 and 35 ($E_5 - A\#_4$). The other events that start the composition are Knight's moves from squares 40 to 23 and 40 to 35, and Bishop's moves from squares 40 to 34 and 40 to 52. This process continues in the same way for the rest of each study. In the seventh and eighth sections, although there are no individual events in the last two files (G and H in chess terminology), for file G the material that remains over time corresponds to the Bishop that starts on file D (fourth section). To carry out the

⁷ The moves are represented in colour: blue for the Knight, green for the Bishop and red for the King. The arrows are placed at the bottom rank of the square used for each study.

process corresponding to file H (eighth section), with durations of 57, 1, 8 and 64 in each study, the solution applied has been the retrogradation of the immediately preceding material during the duration of the section. A global overview of the events with which the composition is built can be seen in APPENDIX 1 (TABLE 1).

1.2. Geometries of movement

Each characteristic move of the Knight, Bishop and King has a specific value. The values are extracted from the diagonals of the moves of the Bishop and King and the move in “L” of the Knight, applying the Pythagorean theorem, as well as from the vertical and horizontal moves of the King. The values corresponding to each move are **2** and **2.83** for the King, **3.61** for the Knight, and **2.83**, **4.25**, **5.66**, **7**, **8.5**, **9.87** and **11.33** for the Bishop (see APPENDIX 2: FIGURE 3).

Dividing the number corresponding to the initial square of each pitch event by the value assigned to the characteristic moves of the Knight, Bishop and King, the number of individual appearances that must take place in the time assigned by the bottom rank is obtained. For example, in the Knight move between the numbers 40 and 23 ($D\#_5-A\#_3$), number 40 is divided between 3.61 to obtain the number of 11 separated actions in $64/3.61=17.75$ quarters (see TABLE 2). In this manner, the distance between the pitches ($D\#_5$ and $A\#_3$), which could be performed continuously, is gradually filled with isolated musical actions, in this section “pizzicati”. The pitch conversion of the numerical results of the division is not restricted to semitones, but quarter tones also appear. For this reason, they appear frequently in the work. An example of this treatment can be seen at the beginning of each study, which is made of different types of pizzicato derived from the events started in the first file. In the work, neither the quantification nor the structure affects the dynamics of the events, although the dynamics are related from one study to another.

At the beginning of each study, five different events appear. Two of them correspond to the duration of the first section which in the first study lasts 64 quarters, while the other three extend to the following sections: 64+2 in two of them and 64+2+3 in one (TABLE 2). In all the events the material used for the first section consists of different types of pizzicato. In the first study “pizzicato

sul ponticello” on Violin I, Viola and Violoncello, “pizzicato ordinario” on Violin II, and “pizzicato with fingernail” played alternately between Violin I and Violin II (FIGURE 4). Each event performs a set number of individual actions until completed, as is shown in TABLE 2 (move’s quantification).

TABLE 2 - (A. Hortigüela), *Chess Studies*, n. 1. Events of the first section.

INSTRUMENT	MATERIAL	DURATION	MOVE'S QUANTIFICATION	SQUARES	PITCHES
Violin I	Pizz. sul ponticello	66 (64 + 2)	41/3.61~11 actions in 66/3.6~18.33	41 - 35	E ₅ - A _{#4}
Viola	Pizz. sul ponticello	66 (64 + 2)	40/3.61~11 actions in 66/3.6~18.33	40 - 35	D _{#5} - A _{#4}
Violoncello	Pizz. sul ponticello	64	40/3.6~11 actions in 64/3.6~17.75	40 - 23	D _{#5} - A _{#3}
Violin II	Pizz. ordinario	64	40/2.8~14 actions in 64/2.8~ 22.87	40 - 34	D _{#5} . A ₄
Violin I / Violin II	Pizz. with fingernail	69 (64 + 2 + 3)	40/5.6~7 actions in 69/5.6~12.33	40 - 52	D _{#5} - D _{#6}

FIGURE 4 - (A. Hortigüela), *Chess studies* (2020), n. 1, Bars 1-3. Different types of Pizzicato at the beginning.

In the second study, the first section lasts 57 quarters. In it, the events that extend to the following sections are of a duration of 57+16 and 57+16+24 (TABLE 3). The “pizzicato with fingernail” and the “pizzicato ordinario” appear with a different instrumental combination:

“pizzicato with fingernail” is played by Violin II, and “pizzicato ordinario” is played alternately by Violin I and Viola (FIGURE 5).

TABLE 3 – (A. Hortigüela), *Chess Studies*, n. 2. Events of the first section.

INSTRUMENT	MATERIAL	DURATION	MOVE'S QUANTIFICATION	SQUARES	PITCHES
Violin I	Pizz. Sul ponticello	57	60/3.61~16 actions in 57/3.6~15.83 s.	60 – 54	B ₆ – F ₆
Viola	Pizz. Sul ponticello	57	60/2.8~21 actions in 57/2.8~20.35 s.	60- 12	B ₆ – B ₂
Violoncello	Pizz. Sul ponticello	73 (57 + 16)	3/3.61~1 actions in 73/3.6~20.27 s.	3 – 20	D ₂ – G ₃
Violin II	Pizz. With fingernail	73 (57 + 16)	60/3.61~16 actions in 73/3.6~ 15.83 s.	60 – 20	B ₆ - G ₃
Violin I / Viola	Pizz. Ordinario	97 (57 + 16 + 24)	60/5,6~10 actions in 97/5.6~17.32 s.	60 – 26	B ₆ – C _{#4}

FIGURE 5 – (A. Hortigüela), *Chess studies* (2020), n. 2, Bars 1-3. Types of Pizzicati at the beginning.

In the third study, there is a strong contrast between the duration of the first section (1 quarter) and that of the second and third sections (63 and 62 quarters respectively). Due to its short duration, the pizzicati corresponding to the first section played by Violin I and Viola are played as "pizzicato glissando". These instruments immediately perform the material corresponding to the second section whose musical material is the "glissando" (FIGURE 6). The events extending to the second section are "pizzicato sul ponticello" in the Cello and "pizzicato with fingernail" in the Violin II, lasting 1+63 quarters. The event that extends to the third section is "pizzicato ordinario" and is played alternately between Violin II and Cello during 1+63+62 quarters (TABLE 4).

TABLE 4 - (A. Hortigüela), *Chess Studies*, no. 3. Events of the first section.

INSTRUMENT	MATERIAL	DURATION	MOVE'S QUANTIFICATION	SQUARES	PITCHES
Violin I	Pizz. Gliss. sul ponticello	1	25/3.61~7 actions in 1/3.6~0.27 s.	25 - 42	C ₄ - F ₅
Viola	Pizz. Gliss. sul ponticello	1	25/2.8~9 actions in ½.8~0.35 s.	25- 31	C ₄ - F ₄ [#]
Violoncello	Pizz. sul ponticello	64 (1 + 63)	24/3.61~6 actions in 64/3.6~17.77 s.	24 - 30	B ₃ - F ₄
Violin II	Pizz. with fingernail	64 (1 + 63)	25/3.61~7 actions in 64/3.6~ 17.77 s.	25 - 30	C ₄ - F ₄
Violin II / Violoncello	Pizz. ordinario	126 (1 + 63 + 62)	25/5.6~4 actions in 126/5.6~22.5 s.	25 - 13	C ₄ - C ₃

FIGURE 6 – (A. Hortigüela), *Chess studies* (2020), n. 3, Bars 1-5.

In the fourth study, the first section lasts 8 quarters. In it, only three of the five starting events are used, all of them as "pizzicato sul ponticello" (TABLE 5). This simplification of events has been chosen due to the brevity of the section and the relatively high density of the appearances of the pizzicati in Violin I and Cello. In turn, the other event that intervenes requires the joint participation of Viola and Violin II, since its first pitch can only be played by the Viola (FIGURE 7). The other two events that should start in this first section, which are "fingernail pizzicato" and "pizzicato ordinario", appear after the first 8 quarters combined with the "glissando" material of the next section.

TABLE 5 - (A. Hortigüela), *Chess Studies*, no. 4. Events of the first section.

INSTRUMENT	MATERIAL	DURATION	MOVE'S QUANTIFICATION	SQUARES	PITCHES
Violin I	Pizz. sul ponticello	57 (8 + 49)	62/3.61~7 actions in 57/3.6~15.83 s.	62 - 45	C# ₇ - G# ₅
Viola / Violin II	Pizz. sul ponticello	8	5/2.8~2 actions in 8/2.8~2.86 s.	5 - 53	E ₂ - E ₆
Violoncello	Pizz. sul ponticello	8	5/3.61~14 actions in 8/3.6~2.22 s.	5 - 11	E ₂ - A# ₂

FIGURE 7 – (A. Hortigüela), *Chess studies* (2020), n. 4, Bars 1-3

1.3. Compositional treatment from each side of the magic square

The translation in time of the numerical structure of the magic square combined with the quantification of the moves of the pieces is the basic element on which the compositional treatment of the work is based. For each formal section, different musical materials have been assigned. Its duration is preset in each study by the numerical sequence of the lower rank. The variation of the magic square in each of his four perspectives, using the same type of material for each section, produces fragments related to each other between the four studies but with different musical evolutions. The appearance of materials with differing temporal structures indicates that the development in each study presents differences in formal and sound aspects. An example of this can be seen in how the differences in duration of the first section influence the treatment of the pizzicato material. As each study develops, the changes in the durations of each new section reproduce this fact generating formal changes between the studies.

The first section of studies 1 and 2 contrasts in duration with that of 3 and 4, and therefore in its formal perception. There are also contrasts in the duration of the following sections in different combinations between the studies, giving rise to a development of material that varies throughout

the work. In the eighth section for example, in which there are no events, a retrogradation of the immediately preceding material is produced during the time indicated for each study (57, 1, 8 and 64 quarters). As in the first section, the contrasts in the duration of retrogradation between the different studies are patent. The development of the durations of the sections and the materials that are put into play is shown in detail in APPENDIX 3: TABLE 6.

2. Musical result

Once the chess material used as a base has been analysed, as well as the conditions under which it is transferred to the environment of the composition, it is necessary to observe the singular structural evolution of each study. Given that the duration of the four studies is the same and the type and order of appearance of the materials used are also similar, the differences between the studies depend on the musical variation derived from the changes in the numerical matrix of the magic square. This fact influences the field of temporal organisation, the transformation of pitches and its effect on the general sonority. As mentioned, each study is structured into 8 sections whose duration is extracted from the number sequence of the first and last ranks (studies 1 and 3) and the last and first files (studies 2 and 4). To complete the form, in the eighth section, which does not contain any event, the immediately preceding material has been retrograded during the duration assigned to the section.

The sequence of moves of the pieces forms the basis of the composition. By superimposing it on each side of the magic square for each study it is possible to observe underlying differences in the numerical constitution of each section throughout the work. Considering, in turn, that the type of materials assigned to each of the sections is identical, a general assessment of the possibilities of inherent musical variation offered by the combination of the moves of the pieces on the numeric matrix can be made (see APPENDIX 3: TABLE 6).

2.1. Comparison between studies

In the section dedicated to movement geometries (1.2), the initial fragments of each study have been analysed. It was possible to observe how the different numerical quantifications of the magic square for each study have a decisive influence on the duration and development of the material used. Thus, significant formal changes can be seen between different studies. Studies 1 and 2 develop the pizzicato material of the first section stably for a certain time. However, because in studies 3 and 4, the first section is short, although the pizzicato material is exposed, it soon merges with the glissando material of the second section. There is thus a clear differentiation in the formal development of the beginning between the pairs of studies 1-2 and 3-4. This formal difference has not been achieved through compositional tools alien to the idea from which the work arises. The original idea is deducing the temporal and pitch relationships from the pure quantification of the moves of the pieces in combination with the values indicated in the magic square matrix.

In addition to those at the beginning, other examples of formal differentiation between studies can be shown. There are also important differences in duration in the second and third sections that already appear between the first and second studies. In the first study, the second section lasts only 2 quarters and the third section only 3 quarters, while in the second study, the second section lasts 16 quarters and the third section 24. The duration of these sections in the third and fourth studies is much longer: 63 and 62 quarters for each section in the third study and 49 and 41 in the fourth. The materials used are "glissando" in the second section and "long note" in the third. The shortness of the third section in the first study results in the long note being played as a scale (FIGURE 8). In the second study, both sections develop their respective materials in a more natural way (FIGURE 9).

FIGURE 8 – (A. Hortigüela) Chess studies (2020), n. 1, Bars. 15-17. Second and third sections.

FIGURE 9 - (A. Hortigüela) *Chess studies* (2020), n. 2, Bars. 13-30. Second and third sections.

Further formal differences inherent to each study can be found by comparing the durations of other sections, for example, the fourth and fifth. In the first study, they are both of very long duration: 61 and 60 quarters respectively. Their durations are also considerable in the second and fourth studies: 33 and 25 quarters in the second and 32 and 40 in the fourth. However, in the third study, the duration of these sections is short: 4 and 5 quarters. The new material used in the fourth section is “tremolo” and in the fifth is “staccato”. The formal compression seen at the beginning of the third study occurs again between the fourth and fifth sections (FIGURE 10). Meanwhile, in other studies, such as the first, second and fourth, the musical material develops more spaced: for example, the staccato material in the fifth section develops as “staccato jeté” (FIGURE 11).

FIGURE 10 – (A. Hortigüela) *Chess studies* (2020), n. 3, Bars. 29-34. Fourth and fifth sections.



FIGURE 11 - (A. Hortigüela) *Chess studies* (2020), n. 2, Bars. 37-39. Fragment of the fifth section.

The last sections also offer important differences in their formal articulation. The new materials brought into play in each of them are “chord” in the sixth section, “long note” in the seventh, and “retrogradation” of the immediately preceding material in the eighth. TABLE 7 shows the evolution of the durations of each study in the final part. The numerical arrangement, translated into temporal space, shows differences between the pairs of studies 1-4 and 2-3.

The sum of the sixth and seventh sections is relatively short in the first and fourth studies: 13 quarters in the first and 26 quarters in the fourth. Its retrogradation, however, is much longer: 57 and 64 quarters respectively. This means that when the material is retrograded at the end of these studies, the predominant materials are the “staccato jeté” and the “long notes” corresponding to the fifth section (FIGURE 12). On the other hand, the process in the second and third studies is reversed. By them the sum of the sixth and seventh sections is very long: 104 quarters in the second and 117 in the third, while its final section retrogrades only 1 quarter note in the second study and 8 quarters in the third. It can be said that in these studies the retrograde ending is formally irrelevant (FIGURE 13). Globally, this final part shows clearly how the translation of the numerical matrix of the magic square influences the formal structure.

TABLE 7 - (A. Hortigüela), *Chess Studies*. Compared durations of sections 6, 7 and 8.

	SECTION 6	SECTION 7	SUM (6+7)	SECCIÓN 8 (Retrograde)	SUM
STUDY N. 1	6	7	13	57	13 + 57 = 70
STUDY N. 2	48	56	104	1	104 + 1 = 105
STUDY N. 3	59	58	117	8	117 + 8 = 125
STUDY N. 4	17	9	26	64	26 + 64 = 90

FIGURE 12 - (A. Hortigüela) *Chess studies* (2020), n. 1, Bars. 48-59. Sixth and seventh sections and the beginning of retrogradation.

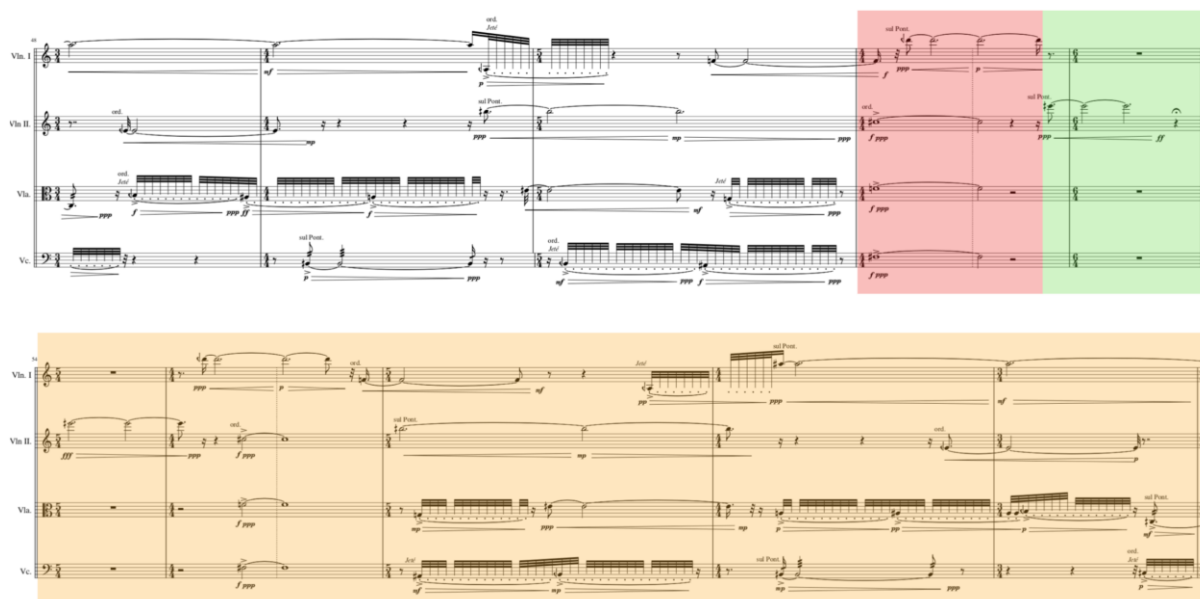


FIGURE 13 - (A. Hortigüela) *Chess studies* (2020), n. 2, Bars. 61-70. Seventh section and the beginning of retrogradation.

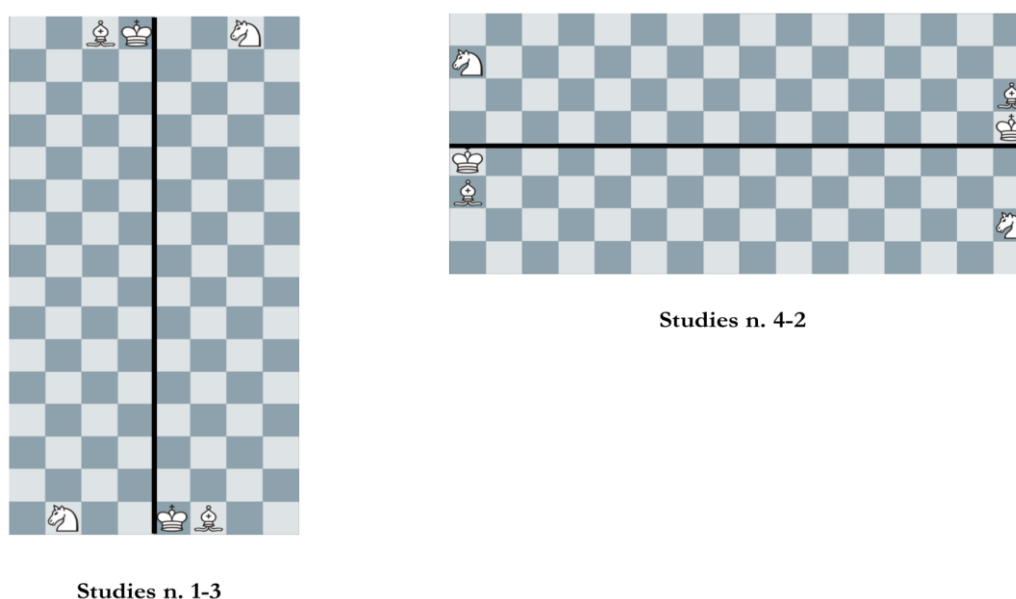


2.2. Interval Symmetry and Chirality

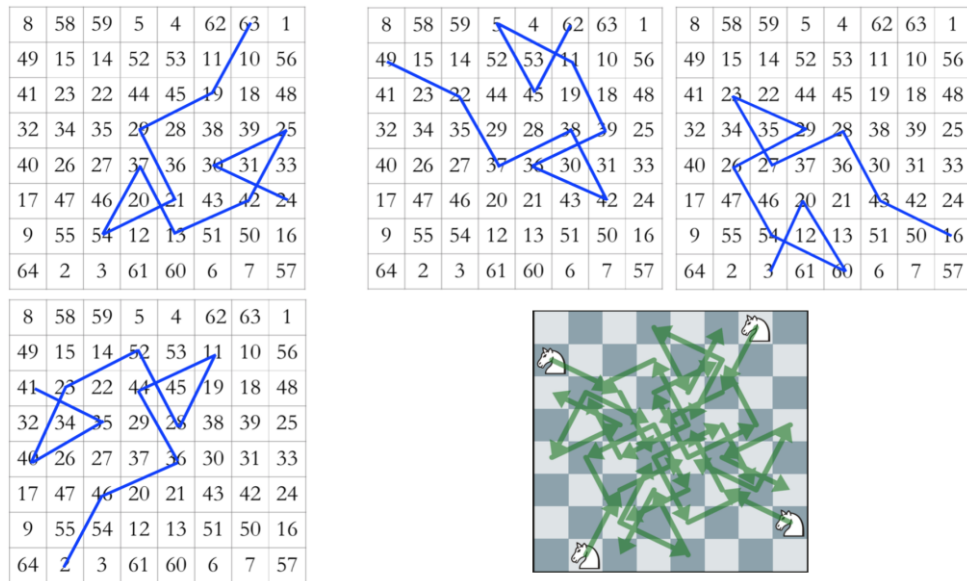
TABLE 7 shows a phenomenon of mirror symmetry between numbers. The relationship between the numbers 6, 7 and 57, corresponding to the sixth, seventh and eighth sections of the first study, is reflected in the third study, whose quantities for these sections are 59, 58 and 8. The symmetry of the chess moves of these pair of studies, as well as those of the second and fourth, responds to the concept of "chirality". Chirality, or chiral asymmetry, is a property that means that a

geometric object cannot be exactly superimposed on its mirror image. The most common example of chirality is that of the hands. Although one of the hands is the mirror of the other, it cannot be superimposed exactly. The concept of chirality appears in various fields of science, such as particle physics, where chirality has to do with the movement of particles in a dextrorotatory or levorotatory direction: clockwise or counterclockwise (Hegstrom, Kondepudi, 1990). Chirality is an intrinsic quality of chess since the same sequence of moves on each side cannot be exactly superimposed on the board. If the initial position of the pieces that perform the sequence of movements used in the work is observed, two chiral pairs can be seen, those corresponding to studies 1 and 3 and 2 and 4 (FIGURE 14). If the sequence of individual moves of the pieces is superimposed on the magic square a chiral relationship appears between the numbers following the same pairs of studies, as shown in APPENDIX 4 (FIGURE 15).

FIGURE 14 - Chirality of the initial position on the vertical and horizontal axis.



The internal layout of the matrix of the chosen magic square manifests numerical chirality in the moves made by the three pieces that produce the sequence, as can be seen in the moves of the Knight (FIGURE 16). Thus, due to their intrinsic chirality, the relationships between pitches are proportional between study pairs 1-3 and 2-4 (TABLE 8).

FIGURE 16 - (A. Hortigüela), *Chess Studies*. Chirality of the moves of the Knight.TABLE 8 - (A. Hortigüela), *Chess Studies*. Numerical chirality of the Knight.

STUDY N. 1	STUDY N. 3	RESULT	STUDY N. 2	STUDY N. 4	RESULT
2-46	63-19	(44)	16-43	49-22	(27)
46-36	19-29	(10)	43-28	22-37	(15)
36-44	29-21	(8)	28-27	37-38	(1)
44-11	21-54	(33)	27-23	38-42	(4)
11-28	54-37	(17)	23-29	42-36	(6)
28-52	37-13	(24)	29-26	36-39	(3)
52-23	13-42	(29)	26-54	39-11	(28)
23-40	42-25	(17)	54-60	11-5	(6)
40-35	25-30	(5)	60-20	5-45	(40)
35-41	30-24	(6)	20-3	45-62	(17)

The treatment of pitches during the work is sometimes altered by the individual duration of the sections and by the quantification of the events within them. Sometimes the sum of the specific durations of the events does not result in the exact pitch that should be reached. For this reason, the exact proportionality between intervals appears in some cases slightly altered. These deviations can be observed by comparing the sixth and seventh sections. The sixth section presents a "chord" whose pitch relationship is proportional between study pairs 1-3 and 2-4. However, in the seventh section, the proportionality of the pitches is not exact. The event of this section is the prolongation of the

"long note" material started in the fourth section, which corresponds to a Bishop move. In the first study, the pitch that should be reached is G_6 , corresponding with the sound pitch 56 (see FIGURE 1). However, due to the quantification of the individual events, only the pitch $E_6 + 1/4$ is reached. On the other hand, its chiral pair, the third study, must come until the pitch $G\#_2$, corresponding to the number 9, which is the pitch reached. Despite this small deviation, due to the rounding of the numerical quantification of the individual events, a mirror structure of the general movement of pitches between these pairs appears clearly towards the high register in the first study and towards the lower register in the third (TABLE 9).

TABLE 9 - (A. Hortigüela), *Chess Studies*. Proportionality of pitches of sections 6 and 7.

	STUDY N. 1	STUDY N. 3	STUDY N. 2	STUDY N. 4
SECTION 6	$F\#_3/F_4/C\#_5$	$D_4/A\#_4/A_5$	$A_3/G_5/G\#_5$	$G_3/G\#_3/F\#_5$
	STUDY N. 1	STUDY N. 3	STUDY N. 2	STUDY N. 4
SECTION 7	$D_6 - 1/4$ up to $E_6 + 1/4$	B_3 up to $G\#_2$	$C\#_6$ up to $A\#_6$	A_2 up to G_2

From TABLE 9 it is possible to extract some considerations about the pitches corresponding to the seventh section. Focusing on the pair of studies 1-3, the pitch movements are $D_6 - 1/4 - E_6 + 1/4$ in the first study and $B_3 - G\#_2$ in the third. You can see the mirror movement of pitches but the intervals in this passage differ. In the first study, the distance is an ascending minor third, while in the third study, the distance is an octave plus a minor third, in this case descending (FIGURE 17). In the other pair, there is also a clear interval difference: in the second study, the pitches $C\#_6 - A\#_6$ are separated by an ascending major sixth, while the pitches $A_2 - G_2$ of the fourth are separated by a descending major second.

FIGURE 17 - (A. Hortigüela) *Chess studies* (2020), n. 3, Bars. 46-68. Event of the Bishop in the seventh section.

The image displays a musical score for a string quartet, specifically for the event of the Bishop in the seventh section of 'Chess studies' (2020), n. 3, Bars. 46-68. The score is written for Violin I, Violin II, Viola, and Violoncello. It is divided into two systems. The first system covers bars 46-68, and the second system covers bars 69-80. The score is written in 4/4 time and features various musical notations including notes, rests, and dynamic markings (f, p, pp, ppp). The event of the Bishop is highlighted in green.

The explanation for this phenomenon must be found in the influence of the temporal factor of each section on the relationships between the pitches. While in the first study, the section lasts only 7 quarters, in the third study this section lasts 58. On the other hand, in the second study, the section lasts 56 quarters, while in the fourth only 9. The final section also shows this divergent parity: 57 and 8 quarters in the first and third studies, and 1 and 64 in the second and fourth studies. Therefore, the numerical matrix, in a natural way, produces a different parity between the studies (in this case between 1-4 and 2-3 instead of 1-3 and 2-4), whose influence is projected onto the form. Also, the last section, which will be retrograded, will follow this new parity, making the last section in the first and fourth studies much more important in terms of form than in the second and third studies (FIGURE 18).

FIGURE 18 - (A. Hortigüela) *Chess studies* (2020), n. 4, Bars. 48-68. Sixth and seventh sections and the beginning of retrogradation.

The intrinsic chirality of the chess material that serves as the basis for the composition implies a high level of organisation. The parity between studies generated serves as a unifying element that gives the work a strong internal coherence. However, the chirality between pairs of studies is nuanced when combined with the organisation of the internal temporality of each one. The different numerical constitutions of each perspective of the magic square mean that the form and the relationships between pitches can change. The parity between studies is subject to transformation. This fact makes each study feel unique, as well as intimately related to the others.

3. Conclusions

The work *Chess Studies* arises from the need to establish distance concerning the treatment of musical materials in the composition process. In this sense, mathematics as a basis for composition is an appropriate means for it. This fact is not a new phenomenon in composition since mathematical concepts such as combinatorics, indeterminacy or topology have been at the core of the compositional conception of different composers. However, although the systematic use of the mathematical possibilities of chess follows this tradition, it represents a new way of approaching the

subject. The differential fact of the proposal is the possibility of adapting the wide range of numerical and geometrical treatments that derive from chess to a wide range of treatments of musical material. These treatments can be adapted to different languages, techniques, and aesthetics. The work presented represents a first approach to those possibilities in contemporary aesthetics and technique.

As a result of the analysis of the composition process some particularities of the effect of the numerical treatment of the magic square, which represents the board, combined with the moves of the pieces, have been revealed in the structure of the work. One of the particularities is the transformation of the formal structure experienced by the different studies, merely based on the change of perspective of the magic square. Time evolves differently from one study to another showing differences and similarities between them. It has been shown that the strong contrasts in the duration of the sections between studies lead to clear structural differences. With this analysis, it has been tried to show how the duration of the sections in this set of four studies projects differences in sound and time evolution. But some similarities provide the set with a perception of unity given mainly by two factors: the use of the same musical materials in the different sections in the four studies, and the intrinsic chirality of the chess material that serves as the basis of the composition.

The order of appearance of the musical materials used in the different studies is the same. When applying them at a specific moment on the score, some compositional decisions have caused a certain material to appear in a study during the totality of the duration of the corresponding event, while only partially in another study. In the places where an ellipsis of a certain material has been chosen, compositional decisions have been made concerning the overlap of materials and the duration of the section. On the other hand, where some appear abruptly, such as the chord in the sixth section, the effect produced on the composition clearly shows the sound evolution structured in blocks, internally differentiated in each study. As a result, the composition, divided into eight sections, is perceived in broader temporal stretches different from one study to another. Despite the restrictions imposed when dealing quantitatively with temporal and pitch material, the sound evolution of each study is individual and organic.

Another fact that has been observed after accurately following the mathematical structure during the process of the composition, is the ability to create fragments with tendencies towards

symmetry between different studies. This feature can best be appreciated in the final stretch of each study. There is also a tendency towards symmetry in the field of pitches: between studies no. 1 and 3 on the one hand and between no. 2 and 4 on the other hand. That is a consequence of adapting the numerical structure of the magic square. As mentioned before, this characteristic symmetry between certain elements of the work can be assimilated into the concept of *chirality*, which shows the sequence of moves of the chess pieces from the different perspectives of the square. The musical translation of this idea of chirality in the numerical field is represented mainly in the sound pitches but also with implications for temporality.

The composition presented here is a first step in the search for perspectives of work that introduces the use of chess as a source for the treatment of different musical elements. In the work, not all the musical variables have been developed from the chess source, but some have. In a higher stage of technical development, other musical variables can be integrated into the possibilities of chess, as well as combined with different technical procedures. It is possible to suggest that the geometry of chess as a basis for the compositional process is not limited to the possibilities of adaptation to a specific work, but that it can be translated into multiple technical points of view that could affect any development of the musical material. The work composed which has been analysed in this paper proves that artistic research in the field of composition represents an open path to broaden the development of compositional technique with new points of view. The beginning of new research on the mathematical possibilities of chess and its adaptation to the environment of musical composition represents the core of this work. Some ideas seen in this article, such as the concept of quantification of the moves of the pieces, the use of the numeric matrix of the board, the symmetry or proportionality of pitches or durations of the parts or the concept of chirality that emanates from chess, are just a few examples of possibilities that research in this field can offer. An investigation of this type can help to understand the importance of numerical and geometrical aspects in the musical phenomenon and its connection with the universe of chess possibilities.

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	Investigation		Visualization
	Methodology		Writing – original draft
	Project administration	X	Writing – review & editing

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APPENDIX 1: TABLE 1

TABLE 1 - (A. Hortigüela), *Chess studies*. Events in the piece.

STUDY N. 1

FILE	MOVE	SQUARES	PITCHES	DURATION
A	Bishop	40 - 34	D# ₅ - A ₄	64
	Knight	40 - 23	D# ₅ - A# ₃	64
	Knight	40 - 35	D# ₅ - A# ₄	66
	Bishop	40 - 52	D# ₅ - D# ₆	69
	Knight	41 - 35	E ₅ - A# ₄	66
B	Knight	2 - 46	C# ₁ - A ₅	2
	Bishop	34 - 13	A ₄ - C ₃	66
	King	23 - 14	A# ₃ - C# ₃	2
	Knight	23 - 52	A# ₃ - D# ₆	5
C	Knight	46 - 36	A ₅ - B ₄	64
	Bishop	27 - 20	D ₄ - G ₃	3
	Bishop	27 - 6	D ₄ - F ₂	124
	Bishop	22 - 52	A ₃ - D# ₆	3
	King	22 - 44	A ₃ - G ₅	3
	King	22 / 14	A ₃ / C# ₃ (simultaneously)	3
D	Bishop	20 - 13	G ₃ - C ₃	61
	Bishop	20 - 56	G ₃ - G ₆	134 (up G-file)
	Knight	44 - 11	G ₅ - A# ₂	121
	King	44 - 45	G ₅ - G# ₅	61
	Knight	44 - 36	G ₅ - B ₄	61
	Knight	52 - 28	D# ₆ - D# ₄	61
E	King	60 / 13 / 21	B ₆ / C ₃ / G# ₃ (simultaneously)	60
	King	21 - 30	G# ₃ - F ₄	60
	King	45 - 19	G# ₅ - F# ₃	60
	Knight	28 - 11	D# ₄ - A# ₂	60
F	King	30 / 38 / 19	F ₄ / C# ₅ / F# ₃ (simultaneously)	6
G	Continuation of the D-file Bishop's move			
H	Retrograde			57

STUDY N. 2

FILE	MOVE	SQUARES	PITCHES	DURATION
A	Bishop	60 - 12	B ₆ - B ₂	57
	Knight	60 - 54	B ₆ - F ₆	57
	Knight	60 - 20	B ₆ - G ₃	73
	Bishop	60 - 26	B ₆ - C# ₄	97
	Knight	3 - 20	D ₂ - G ₃	73
B	Knight	16 - 43	D# ₃ - F# ₅	16
	Bishop	12 - 39	B ₂ - D ₅	73
	King	54 - 47	F ₆ - A# ₅	16
	Knight	54 - 26	F ₆ - C# ₄	40
C	Knight	43 - 28	F# ₅ - D# ₄	58
	Bishop	21 - 30	G# ₃ - F ₄	24
	Bishop	21 - 48	G# ₃ - B ₅	82
	Bishop	46 - 26	A ₅ - C# ₄	24
	King	46 - 27	A ₅ - D ₄	24
	King	47 / 46	A# ₅ / A ₅ (simultaneously)	24
D	Bishop	30 - 39	F ₄ - D ₅	33
	Bishop	30 - 58	F ₄ - A ₆	162 (up G-file)
	Knight	27 - 23	D ₄ - A# ₃	58
	King	27 - 35	D ₄ - A# ₄	33
	Knight	27 - 28	D ₄ - D# ₄	33
	Knight	26 - 29	C# ₄ - E ₄	33
E	King	25 / 39 / 38	C ₄ / D ₅ / C# ₅ (simultaneously)	25
	King	38 - 45	C# ₅ - G# ₅	25
	King	35 - 22	A# ₄ - A ₃	25
	Knight	29 - 23	E ₄ - A# ₃	25
F	King	45 / 44 / 22	G# ₅ / G ₅ / A ₃ (simultaneously)	48
G	Continuation of the D-file Bishop's move			
H	Retrograde			1

STUDY N. 3

FILE	MOVE	SQUARES	PITCHES	DURATION
A	Bishop	25 - 31	C ₄ - F# ₄	1
	Knight	25 - 42	C ₄ - F ₅	1

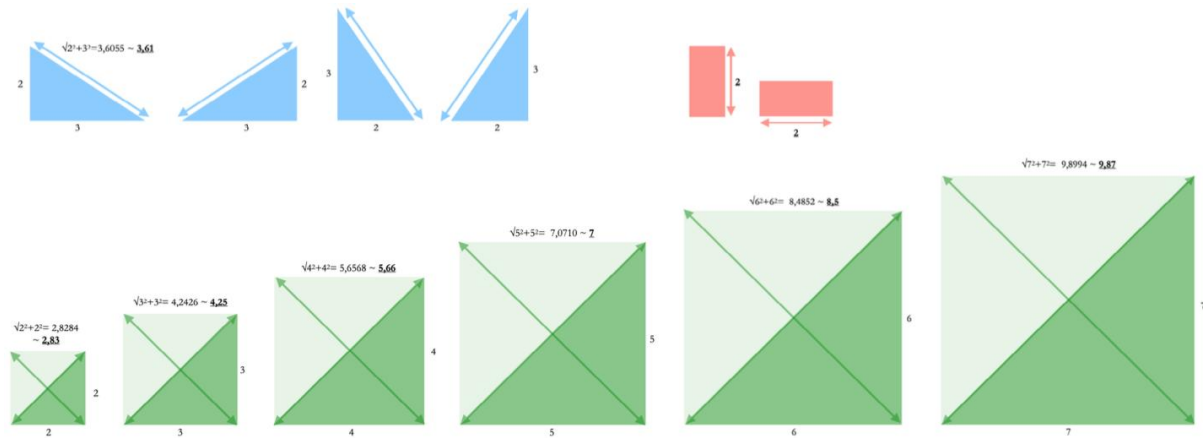
	Knight	25 - 30	C ₄ - F ₄	64
	Bishop	25 - 13	C ₄ - C ₃	126
	Knight	24 - 30	B ₃ - F ₄	64
B	Knight	63 - 19	D ₇ - F ₃ [#]	63
	Bishop	31 - 52	F ₄ [#] - D ₆ [#]	129
	King	42 - 51	F ₅ - D ₆	63
	Knight	42 - 13	F ₅ - C ₃	125
C	Knight	19 - 29	F ₃ [#] - E ₄	66
	Bishop	38 - 45	C ₅ [#] - G ₅ [#]	62
	Bishop	38 - 59	C ₅ [#] - A ₆ [#]	71
	Bishop	43 - 13	F ₅ [#] - C ₃	62
	King	43 - 21	F ₅ [#] - G ₃ [#]	62
	King	43 / 51	F ₅ [#] / D ₆ (simultaneously)	62
D	Bishop	45 - 52	G ₅ [#] - D ₆ [#]	4
	Bishop	45 - 9	G ₅ [#] - G ₂ [#]	126 (up G-file)
	Knight	21 - 54	G ₃ [#] - F ₆	9
	King	21 - 20	G ₃ [#] - G ₃	4
	Knight	21 - 29	G ₃ [#] - E ₄	4
	Knight	13 - 37	C ₃ - C ₅	4
E	King	5 / 52 / 44	E ₂ / D ₆ [#] / G ₅ (simultaneously)	5
	King	44 - 35	G ₅ - A ₄ [#]	5
	King	20 - 46	G ₃ - A ₅	5
	Knight	37 - 54	C ₅ - F ₆	5
F	King	35 / 27 / 46	A ₄ [#] / D ₄ / A ₅ (simultaneously)	59
G	Continuation of the D-file Bishop's move			
H	Retrograde			8

STUDY N. 4

FILE	MOVE	SQUARES	PITCHES	DURATION
A	Bishop	5 - 53	E ₂ - E ₆	8
	Knight	5 - 11	E ₂ - A ₂ [#]	8
	Knight	5 - 45	E ₂ - G ₅ [#]	57
	Bishop	5 - 39	E ₂ - D ₅	98
	Knight	62 - 45	C ₇ [#] - G ₅ [#]	57
B	Knight	49 - 22	C ₆ - A ₃	49
	Bishop	53 - 26	E ₆ - C ₄ [#]	122

	King	11 - 18	A# ₂ - F ₃	49
	Knight	11 - 39	A# ₂ - D ₅	90
C	Knight	22 - 37	A ₃ - C ₅	73
	Bishop	44 - 35	G ₅ - A# ₄	41
	Bishop	44 - 17	G ₅ - E ₃	113
	Bishop	19 - 39	F# ₃ - D ₅	41
	King	19 - 38	F# ₃ - C# ₅	41
	King	19/18	F# ₃ / F ₃	41
			(simultaneously)	
D	Bishop	35 - 26	A# ₄ - C# ₄	32
	Bishop	35 - 7	A# ₄ - F# ₂	98 (up G-file)
	Knight	38 - 42	C# ₅ - F ₅	72
	King	38 - 30	C# ₅ - F ₄	32
	Knight	38 - 37	C# ₅ - C ₅	32
	Knight	39 - 36	D ₅ - B ₄	32
E	King	40 / 26 / 27	D# ₅ / C# ₄ / D ₄	40
			(simultaneously)	
	King	27 - 20	D ₄ - G ₃	40
	King	30 - 43	F ₄ - F# ₅	40
F	Knight	36 - 42	B ₄ - F ₅	40
	King	20/21/43	G ₃ / G# ₃ / F# ₅	17
			(simultaneously)	
G	Continuation of the D-file Bishop's move			
H	Retrograde			64

APPENDIX 2: FIGURE 3

FIGURE 3 - (A. Hortigüela), *Chess studies*. Quantification of the moves of the pieces. Verticals and horizontals from 3 to 8 occur in the same way.

APPENDIX 3: TABLE 6

TABLE 6 - (A. Hortigüela), *Chess studies*. Duration and materials of the sections in each study.

STUDY N. 1	SECTION	DURATION	BARS	MATERIAL
	1	64	1 - 16	Pizzicato
	2	2	17	Glissando
	3	3	17	long Note /Scale (glissando)
	4	61	18 - 33	long Note /Pizz. /Tremolo
	5	60	34 - 50	Staccato /Tremolo /long Note
	6	6	51 - 52	Chord
	7	7	52 - 53	long Note
	8	57	54 - 68	Retrograde
STUDY N. 2	SECTION	DURATION	BARS	MATERIAL
	1	57	1 - 16	Pizzicato
	2	16	17 - 21	Glissando
	3	24	21 - 28	long Note /Scale (glissando)
	4	33	28 - 36	long Note /Pizz. /Tremolo
	5	25	36 - 42	Staccato /Tremolo / long Note
	6	48	43 - 54	Chord

7	56	55 - 69	long Note
8	1	70	Retrograde

STUDY N. 3	SECTION	DURATION	BARS	MATERIAL
	1	1	1	Pizzicato
	2	63	2 - 16	Glissando
	3	62	17 - 32	long Note /Scale (glissando)
	4	4	32	long Note /Pizz. /Tremolo
	5	5	33	Staccato /Tremolo / long Note
	6	59	34 - 50	Chord
	7	58	50 - 66	long Note
	8	8	67 - 68	Retrograde

STUDY N. 4	SECTION	DURATION	BARS	MATERIAL
	1	8	1 - 2	Pizzicato
	2	49	3 - 14	Glissando
	3	41	14 - 24	long Note /Scale (glissando)
	4	32	25 - 34	long Note /Pizz. /Tremolo
	5	40	35 - 48	Staccato /Tremolo / long Note
	6	17	48 - 54	Chord
	7	9	54 - 57	long Note
	8	64	58 - 80	Retrograde

APPENDIX 4: FIGURE 15

FIGURE 15 – (A. Hortigüela), *Chess studies*. Chirality of the individual moves of the pieces.

