Reimagining a Riemannian symbology for the structural harmonic analysis of 19th-century tonal music.

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Abstract: In the context of a preview of central topics taken from a larger research work, this article presents and explains a proposition of a functional analytical symbology which is currently being developed as a tool for the structural harmonic analysis of tonal music. This symbology constitutes a key component for a proposal of an analytical methodology for 19th-century extended tonality, which is the core of the aforementioned larger research work. Based on a critical revision of the history of music theory and analysis, this methodology is mainly the result of the recasting and the amplification of thoughts and concepts developed by 19th-century theorists such as Hugo Riemann and Arthur von Oettingen. The article introduces the proposed analytical symbology through a comparison to its historical counterparts, and it also highlights the ability of the proposed symbols to graph the harmonic language of 19th-century extended tonality by means of a few analytical examples.

Keywords: Music Analysis; Functional Harmony; Analytical Harmonic Symbology; Tonal Harmony.

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The present paper intends to anticipate to the general academic public a proposal of an analytical harmonic symbology for tonal music, specially for its chromatic type from the second half of the 19th century. This symbology is part of a series of propositions of taxonomies, nomenclatures, and analytical symbologies currently being developed as part of a larger research project of formulation of a structural model for 19th-century tonality that aims at finding, explaining, and graphing the constructive coherence of the repertoire of the second half of the 19th century in a clear, synthetic, and pedagogical way, grounded on a methodology based on the critical revision of historical theoretical bibliography of that time. Since the main interest of this research concerns the late Romantic repertoire, the most important sources have been, either directly or mediated by contemporary musicologists, texts from the 19th century and beginning of the 20th century by authors such as Gottfried Weber (1779-1839), Anton Reicha (1770-1836), Moritz Hauptmann (1792-1868), François-Joseph Fétis (1784-1871), Arthur von Oettingen (1836-1920), Carl Friedrich Weitzmann (1808-1880), Arnold Schoenberg (1874-1951), Heinrich Schenker (1868-1935), and specially Hugo Riemann (1849-1919), the original creator of the discipline currently called Functional Harmony, and which is considered one of the most important theorists and music thinkers of the 19th century (see REHDING 2003). Constructed from this critical revision of the 19th-century theoretical imagination, this research intends to propose an alternative way to explain the compositional procedures of the extended tonality, formalized in the form of a structural model which will be presented to the most possible extent as a logical deductive system demonstrated "in geometrical order", as in Spinoza's Ethics, in the most objective and pedagogical way possible. With this research, the goal is to not solely create a pedagogical and theoretical body of texts for the teaching of musical structuring, but to also retrieve for the contemporary scholars part of the musical theoretical imagination of the 19th century, a body of work which is reasonably unknown to the contemporary traditional methodologies of musical instruction, despite its utmost importance for understanding 19th-century repertoire.

While the definitive formalization of this research is being prepared in the form of a treatise, some of its most important aspects are being presented to the academic community separately in the form of individual articles. This present article reworks in English a text of mine which was previously presented in Portuguese in congress proceedings (BITTENCOURT, 2009), here prepared in an expanded, more precise and correct way. The analytical symbological reform here presented, together with a proposal of a renewed taxonomy of modulation (see BITTENCOURT, 2013), form the main basis for the analytical methodology proposal which makes possible this bigger research of a structural model for 19th-century tonality.

1. Reimagining Functional Harmony

Despite the immense and evident achievements of the scale-degree theory and its historical and musicological importance (see BERNSTEIN, 2002, p. 778-788), I am of the opinion that the functional harmony theory is much more agile, direct, and competent to explain the logic of the tonal chord progressions of the Classical and Romantic periods. Nonetheless, this research made it clear that, in the manner in which functional harmony is posited and practiced in books such as the ones by Koellreuter (1980), Cyro Brisolla (2006), Diether de La Motte (1998), Dirk Haagmans (1916), and Zula & Marilena de Oliveira (1978), for example, it is not specially fit to the task of revealing with accuracy and clarity the harmonic pathways of the Late Romantic period. This is not at all surprising if we consider the complexity of that repertoire and the fact that the Riemannian original functional concepts have undergone several layers of revisions and acute simplifications since the beginning of the 20th century (see MICKELSEN, 1977, p. 92). Because of this, the present research has been gradually compelled to return to the original 19th-century and early 20th-century sources, reassembling and reworking the concepts of the functionality of the chords and the logic of the harmonic progressions. This task included even the reconsideration of several ideas which were rather banished during the 20th century, such as the major-minor harmonic dualism, for example (see MICKELSEN, 1977, p. 89 and REHDING, 2003, p. 7-8). The present research was mainly conducted from the original works and concepts by Riemann, and this meant the important necessity to undo part of the revisionist work of functional harmony concepts made by authors such as Hermann Grabner (1886-1969) and his pupils Hugo Distler (1908-1942) and Wilhelm Mäler (1902-1976), whose texts form the theoretical grounds of the functional harmony modalities taught nowadays at universities and conservatories all over the world (see MICKELSEN, 1977, p. 92 and REHDING, 2003, p. 7). Despite the strong revisionist attitude of this research, a great effort is made to ground the symbology proposal on the traditional academic nomenclature of the functional harmony found in the more modern theory texts.

2. Basic characteristics of the analytical symbology

The analytical harmonic symbology here presented was conceived according to a series of requisites and needs established by the envisioned analytical methodology. These needs resulted in the postulation of eight basic symbological traits, which will be explained next. First, the notation must be as clean as possible, without trying to convey excessive and unnecessary amounts of information. Second, it needs to reveal by itself and clearly the results and conclusions from an analytical work, revealing above all the logic behind the use of the chords in the specific contexts of the musical text.

This means that the symbology must not end up by being a sort of musical shorthand, a simple reduction of that which is effectively read in the musical text. The analytical notation must instead be the formalization of an understanding of the contextual effect of a harmony, serving not to label it but to explain its behavior. Nonetheless, the symbology must never be ambiguous: every symbol must correspond to one and only one harmonic structure. Third, the notation must be reasonably compatible to the modern functional harmony symbologies in current use. In spite of that, since several usual academic theoretical viewpoints are questioned or reinterpreted, there will obviously be marked differences in some respects. There must also be a precise conversion method between the proposed symbology and the one of the scale-degree theory, which serves nowadays as a *lingua franca* between the theorists of the world. This guarantees that the use of this new nomenclature will not alienate its user from the rest of the world, but it will instead serve to support an improved comprehension of the musical contexts, a fact which will most certainly have a significant impact in the reformulation of the use of the scale-degree theory and in the reformatting of its symbology (see fig. 9, for example). Fourth, the notation should reveal harmonic entities abstracted according to universal and basic principles, and not according to their historical particularities of use. Thus we avoid inconvenient situations such as the multiple forms of notation used for certain chords in LA MOTTE 1998, a procedure which, despite its undeniable historiographical value, serves only to cloud the understanding of a universal harmonic phenomenon, hiding it under the guises of historical particulars. Fifth, the notation must propose a defined theoretical position on the notorious question of the correspondence between function and scale-degree. In Riemann, this matter is addressed in a deliberately contradictory way in relation to his dualist theory (see MICKELSEN, 1977, p. 62-63), probably due to his deep respect for the usage of certain historical terminologies. In this proposed analytical methodology, the harmonic functions are circumscribed not exactly to specific scale-degrees but instead to the logical tasks of harmonic coherence, in the manner explained by Riemann[2] (see MOONEY, 2000, p. 84-85). The consequence of this stance was the revised addition to the symbology of elements from the Riemannian harmonic dualism. Sixth, in consequence of this aforementioned stance and in a much pondered way, the symbology must try to recover concepts from the 19th-century harmonic dualism. Thus, it will be recast as a duality of polarities between harmonic fields. This implicated in a preference for the terminology by Grabner at the expense of the one by Mäler (see MICKELSEN, 1977, p. 92-94), for although both are of monistic orientation, the former is much closer to the original dualist notation by Riemann. The later, excessively monist, does not use specific signs to indicate modality, just letter-case

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[2] Riemann associates the idea of tonal center (thesis) with the harmonic function of tonic, and the idea of departure of the center (antithesis) with the function of subdominant. The idea of refutation of departure and agent of the returning conduction to the center (synthesis) is associated by Riemann with the function of dominant. These functions assemble in the "*große cadenz*", the I-IV-I in conjunction with I-V-I: the ubiquitous I-IV-V-I (see MOONEY, 2000, p. 84-85).

variations of the basic letters. This characteristic, in my opinion, causes a series of inconveniences, for it makes it difficult to visualize operations of modal borrowing. Seventh, the notation must reflect a theoretical redefinition of the concept of chromaticism, separating the concepts of real chromaticism and harmonic borrowing. For the analysis of extended tonality works, I became convinced of the importance of a theoretical separation between the chromaticism pertaining to operations of harmonic borrowing, which is here called "illusory chromaticism", and that which is here called "real chromaticism", which is the chromaticism pertaining to operations of artificial creation of individual leading tones, without the recourse of actions of harmonic borrowing. Eighth, the proposed nomenclature should not aim at indicating a harmonic structure by itself. First of all, its goal is to graph the basic scheme of polarizations and of natural voice leading of the structure, mapping its logical contextual position inside the musical discourse. It is worth to remark here that it is the basic scheme of natural voice leading that ultimately gives breath to the harmonic functions.

3. Description of the analytical symbology

3.1. Initial remarks

This proposal for an analytical symbology had as point of departure the simplified functional theory proposed in the texts by Brisolla (1979) and Koellreuter (1980), both evidently derived from the revisions by Grabner. The symbological revision proposed by Mäler, whose influence can be seen in texts such as the ones by De La Motte (1976), Oliveira (1978) and Menezes (2002), was here completely rejected by not permitting the clear visualization of a change of modality as a polarity change. From this point on, the process of recreation of the functional symbology was grounded in the readings of theoretical texts from 19th-century authors, specially Riemann's own works, and in analytical observations of historical repertoire, which served as a methodological testing ground. The result of this process is the symbology which will be presented next.

In this proposal, the symbols were planned to be read always in the same way, avoiding interpretation doubts and ambiguities, and their components are classifiable in three basic groups: letters, signs, and numbers. As a rule, one should first read only the letters and the signs associated to them, always in vertical columns starting from bottom to top, and from right to left. After this, one proceeds by reading the numbers and their signs.

3.2. Letters

Faithful to the Riemannian functional tradition, this proposal for an analytical symbology continues to adopt the letters T, S, and D, always in upper cases, to denote the main functions of tonic, subdominant, and dominant, respectively (see RIEMANN, 1903, p. 7-8). These letters can be used alone or combined with the letters "r" (relative substitute) and "a" (anti-relative substitute), always in lower case, which are placed to the inferior right of the upper-case letters of the main functions. These smaller letters serve to notate the two basic types of feigning consonances (*Scheinkonzonanz*), which can serve as substitutive harmonies to the main ones (see RIEMANN, 1903, p. 72), as shown in the examples from figure 1a. These are, respectively, the relative substitutes[3] (*Parallelklänge*), previously notated with a "p" both in Riemann's texts and in Grabner's and with an "r" in the Brazilian texts, and the leading-tone substitutes[4] (*Leittonwechselklänge*), previously notated in Riemann's texts with a < (in major) or a > (in minor) crossing the main letters. These last ones are also known as anti-relative or counter-relative substitutes (*Gegenparallelklänge*) in the revision by Grabner (see MICKELSEN, 1977, p. 92), and they are notated with a "g" in German texts and with an "a" in Brazilian texts such as BRISOLLA 2006 and KOELLREUTER 1980. Figure 1c shows a small comparison between the different historical symbols (for this, see also fig. 9). The symbology proposal also keeps the special Riemannian notations of a double D and a double S for the chords of the dominant-of-the-dominant and the subdominant-of-the-subdominant, respectively (see fig. 1b).

Figure 1. Examples of the symbological use of letters, and a comparison of historical symbologies.

To correct the problem relative to the association of the function of a chord not exactly with a scale degree but with a dialectic stage of a harmonic progression, it was necessary at this point to

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[3] The relative substitute (Riemann's "parallel" substitute) is the perfect triad which results from the substitution of the fifth of the generating note of a perfect triad with the major sixth of its generating note. This generating note is the very root, in the case of a major triad, or the fifth of the root, in the case of a minor triad. Thus, the relative substitute of I is vi, of i is III, and conversely (see fig. 9, and RIEMANN, 1903, p. 71-72).

[4] Riemann's leading-tone substitute is the perfect triad which results from the substitution of the generating note of a perfect triad with its leading tone, according to the modality of the triad: a major, Ionian leading tone, in the case of a major triad, and a minor, Phrygian leading tone, in the case of a minor triad. Thus, the anti-relative substitute of I is iii, of i is VI, and conversely (see fig. 9, and RIEMANN, 1903, p. 80).

include dualist elements to the symbology with the addition of the mirror images of the letters D and S (see figures 2a and 2b) to denote the regnant and the supra-regnant, respectively. The regnant (or inverse dominant), a dualist term by Oettingen (see OETTINGEN, 1866, p. 67), has the same functional properties of the dominant, but with its natural resolutions and voice leading in an inverted "upside-down" way, and it is represented by the triad of iv, always as a minor chord. Its counterpart, the supra-regnant (or inverse subdominant), is represented by the chord of v, also always as a minor chord (ibid., p. 67). The idea of using inverted letters was in a certain way inspired by Karg-Elert's symbology (see MICKELSEN, 1977, p. 90).

Figure 2. Examples of the notation for regnants and supra-regnants.

In fact, both the dominant and the regnant can be shown to belong to the same "hyper-dominant-structure", which could be described as something like a dominant eleventh chord in the context of a minor mode. This means that it is possible to form a hybrid dominant/regnant structure using an incomplete version of the aforementioned hyper-structure. As an exemplification of this, figure 10 shows in its measure 29 this equivalence between the dominant and the regnant, with the harmonic analysis revealing a change of voice-leading focus towards a structure whose behavior imprints a "plagal-like" voice-leading quality to a cadential close in G minor, which as such seems to float between being derived from a V-i or a iv-i.

3.3. Signs added to the letters and to numbers of symbols

In this proposal for an analytical symbology, part of the signs are to be aggregated to the letters and numbers of the analytical symbols in several occasions and contexts to indicate modal polarities (° e +), real chromatic alterations (< e >), the omission of structural members of the harmony (\), the antipode substitute relationship (ψ), a region or tonality (—), a harmonic field of borrowed origin (parentheses), and dualist inversion of intervallic logic (↓).

The first marking to be explained, the inverted solidus, is a cut through the main upper-case letter of the function (see fig. 3a). This marking denotes the omission of the root of the harmonic structure, if the letter is cut only once, and the omission both of the root and its third, if the letter is

parallelly cut twice. This is an original marking by Riemann (see RIEMANN, 1903, p. 70), and in the traditional texts it is usually drawn in the opposite direction of my version: /. By an aesthetic choice, I preferred the other way in order to avoid eventual collisions with numbers placed at the superior right. Basically, this marking serves to notate subsets of dominant seventh and dominant ninth chords, a very old concept of derivation which had already appeared in theoretical works by Heinrich Christoph Koch (1749-1816) (see RIEMANN apud MICKELSEN, 1977, p. 203). An interesting issue is that in the case of regnants (for they are inverted versions of dominants) the inverted solidus denotes the omission of the fifths of their roots, in the usual dualist manner, as can be seen in figure 10, measure 29.

The signs < and > , also used by Riemann, serve to indicate the aforementioned chromatic alteration of the real type. They are placed to the right of a number, denoting the chromatic rising or lowering of its intervals, respectively. These signs are mainly used in connection to the chromatic rising or lowering of the fifth of the dominant harmony (see figure 3f). In some extremely rare cases, it is possible to also extend this possibility of real chromaticism to the seventh of the dominant (which is the fifth of the generating tone of the regnant).

The next two signs, ° e + , serve to indicate the borrowing of elements from the harmonic field of inverse polarity, or rather, the region of the modal parallel. These signs must be read respectively as "parallel minor" and "parallel major". Since Riemann[5], the signs ° and + have been adopted by the functional harmony with the meaning of minor and major, and this continues to be used by its Grabnerian branch, which is what we see in modern texts such as the ones by Koellreuter (1980) and Brisolla (2006). Riemann used those signs in a dualist manner, with the symbol for minor placed to the left of the letters, and the one for major placed to the right (see fig. 1c). In the texts by Grabner, Brisolla, and Koellreuter, these signs are always placed to the left of the main letter. Something I find interesting about the use of these signs for minor and major is that they show the modality of a triad in a visual way, almost like an indication of electric polarity. In opposition to this practice, both the traditional notation by Weber via roman numerals and the functional notation by Mäler trust the indication of modality to the cases of the letters (see LA MOTTE, 1998, p. 74), as seen in figs. 1c and 9. In a certain way, this practice seems to me to represent in an iconographic manner a monist concept of modality, with the minor world being characterized as a simple variant of the major. During the development of this symbological proposal, I felt the need to resuscitate the 19th-century dualism not in the way it was originally conceived but as a dialectics of two harmonic fields. This duo is formed by the major harmonic field, here observed with its specific distance relationships and its polarity fluxes,

and its isometric transformation by reflection, in other words, its "upside-down" inverted double, which is the minor harmonic field. This one possesses, as consequence of the very process of reflection, identical distance relationships to the major field, although in an inverted way and with the opposite polar orientation. In this symbology proposal, the return of the harmonic dualism meant initially the necessity of maintaining the Riemannian manner of notation, rejecting the one by Mäler. Nonetheless, while in Oettingen's and Riemann's texts these signs serve to indicate individually the polarities and the essentially constructive orientations of a triad (from bottom to top in the major triad, and vice-versa in the minor), in the present symbology the signs serve to indicate the harmonic field of origin of a harmony, understood with its implications of polarity. According to this, the signs ° and + are used in two different positions: to the superior left of the main letter and to the right of the main letter (see figs. 3b and 1a). The difference of meaning between these cases is due to the reading rules mentioned earlier (item 3.1.). For example, Tr+ means, in full and literally, "the major parallel of the relative substitute of the tonic"; °Tr means "the relative substitute of the tonic of the parallel-minor region"; °Tao means "the minor parallel of the anti-relative substitute of the tonic of the parallel-minor region" (see fig. 3b). In order to simplify, we can omit the modal signs of the harmony symbols if the main tonality is well marked with its modality indication at the beginning of the analysis (see fig. 1a), or also if the harmonic structures belong to the context of a tonality which is well indicated as a region in the analysis. A consequence of the direction-reading rules is noticed in the notation for dominants, which will be reasonably different than the usual functional notations. Because a dominant always includes a leading tone no matter the modality of the key (since the harmony of v, by lack of polar efficiency, cannot operate functionally as a dominant), there is no difference between a structure marked as +D ("the dominant of the parallel major region") and one marked as °D ("the dominant of the parallel minor region"), for in both occasions these are major triads, therefore coincident. The same occurs with the inverse dominants (the regnants) of the modal parallels, which are always minor and therefore always coincident. If we wish to identify the harmony of v as a parallel minor version of a D, we will indicate that with a D°: "the minor parallel of the dominant" (see fig. 3d). The most important usefulness of these coincidences is in the identification of the nature of the ninths of a dominant, which will never be marked with the signs < and > for chromatic alteration. In the present theoretical proposal, this analytical subtlety is due to the importance of the theoretical separation between modal borrowing and real chromaticism. In this view, the major or minor nature of a ninth of a dominant is solely due to the modality of the harmonic field to which the dominant belongs, and an eventual modal

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[5] It is important to remark that these same signs (° and +) are also used in the traditional scale-degree theory with the very different meaning of diminished and augmented, respectively. Historically, the sign ° was already used to mean diminished since the symbology proposed by Gottfried Weber (see RIEMANN apud MICKELSEN, 1977, p. 207). As far as the signal + is concerned, its use was added by Friedrich Richter in 1853 (ibid., p. 211). According to Riemann, Oettingen was the one

borrowing will be correspondingly indicated as being an imported harmony from the harmonic field of opposite polarity. As an example, if in the context of a major key we borrowed the dominant ninth chord of its parallel minor harmonic field (which naturally possesses a minor ninth), we would indicate that occurrence as °D9 and not as D9o or even D9> (see figures 3a and 3c). In another example, seen in figure 10, measure 22, a + sign is used to the left of the symbol for a dominant-of-the-dominant structure to stress the presence of the ninth from the major harmonic field in a structure with a chromatically-lowered fifth, which usually appears only in the context of minor keys.
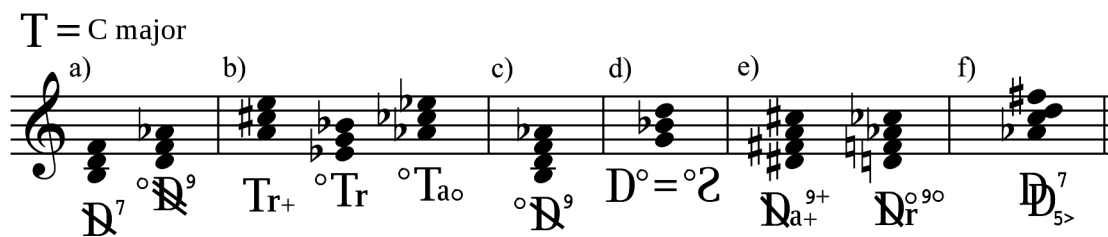

Figure 3. Examples of the use of signs added to the letters and numbers.

Only in exceptional cases (but still with the connotation of an inversion of modal polarity and not of a real chromatic alteration) we will indicate the major ninth with a 9+ and the minor ninth with a 9o, whenever there is the need to indicate a special modal borrowing, specially in cases of extraordinary dominants (see figure 3e). In this theoretical proposal, an extraordinary dominant is a substitute harmony of the main dominant[6], which is constructed and effectively connected to the tonic harmony according to procedures based on the voice-leading schemes found in the most common cadences, specially the typical traditional deceptive cadences and the cadential six-four. Figure 4 shows a basic inventory of these extraordinary dominants and their natural voice-leading paths, which can still be further appended by varying those structures by means of the combined action of the addition of a major or minor ninth, of real chromatic alterations in the fifths, and also of omission of roots and thirds. Also, a "real-life" example of the use of extraordinary dominants can be seen in the analysis contained in figure 10.

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responsible for the association of these same signs with the meanings of minor and major (ibid., p. 219).

[6] Of great importance for the study of the repertoire of the second half of the 19th century, the concept of this collection of accessory dominants is inspired by ideas for unorthodox interconnections between chords via basic models of voice leading suggested by Karl Friedrich Weitzmann in his *Harmoniesystem* of 1860 (see RUDD, 1992, p. 65-69). As a matter of fact, such connections also deserved Schenker's attention (see the cases of tonicalization in SCHENKER, 1954, p. 265-276).

Figure 4. Basic inventory of the extraordinary dominants of the region of C (major and minor).

In the way proposed, the combination of the signs for chromatic alteration and modal borrowing comes to tame the conflicting profusion of signs used for the same harmonic functionality by Riemann and subsequent theorists, an unnecessary complication given my concerns for the observation of the functional phenomena in their universality, reducing all particular versions to one single common model of usage. For example, to indicate an eventual borrowing of the harmony of I in the context of a minor key region, Riemann used in several occasions (see REHDING, 2003, p. 55) the symbols T+ (major tonic), °T3< (minor tonic with a raised third), and °Tv (variant of the minor tonic). In the present proposal for an analytical symbology, such particularities get reduced simply to +T ("tonic of the parallel major region"), a choice which recognizes the use of modal borrowing as the universal source of all those particular contexts.

The sign ↓ , a dualist atavism in my theory, is rarely used and it is connected to the inverse dominant, the regnant. When used, it is written to the superior right of the main letter of the function and immediately before the numbers indicating dissonances, meaning that the marked intervals are to be calculated downwards from the fifth of the basic triad (see figure 2b). This detail represents the most radical infiltration of the Riemannian harmonic dualism in the present proposal for an analytical methodology, and because of this it is most modestly used and only for the sake of demonstrating some important functional equivalences of the extended tonality. An example of this kind of equivalence can be seen in figures 8c and 8d. In 8c, the same chromatic progression of three chords (taken f is seen according to two different points of view, one of them using dualist concepts. In 8d, we see a dualist explanation for a progression which includes a harmony derived from an augmented-sixth chord. Already commented earlier, measure 29 from figure 10 also demonstrates the usefulness of this marking.

The sign ⏤ , placed immediately above a main functional letter, indicates that the symbol does not refer to a harmonic structure but it instead indicates and names a tonal region, thus incorporating Schoenberg's theory of regions[7] (see SCHOENBERG, 1969, p. 20) to the functional harmony (see figure 5b and figure 6). More than a simple expedient to name regions, this symbology serves to map the positioning, the distances, and the kind of the relationship between regions. To exemplify this idea, the table from figure 6 shows the indications of the relationships between the region of C major and all the other regions, measured in the most direct way possible.

a)

$$\underset{(Sr)}{D} = \text{harmonic borrowing of the harmony of V from the harmonic field of the relative subdominant region}$$

b)

$$\underset{(D)}{\overline{Tr}} = \text{region of the tonic relative in the context of an area centered on the region of the dominant of the main key}$$
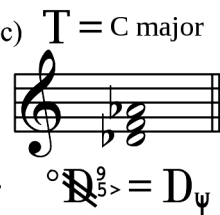
Ex: region of F# minor, if T=D major

c)

$$T = \text{C major}$$

$${}^\circ N^9_{5>} = D_\psi$$

Figure 5. Examples of the indication of harmonic borrowings, regions, and the antipode substitute.

	C	D♭	D	E♭	E	F	F#	G♭	G	A♭	A	B♭	B
M	⁺T̄	ᵒS̄a	D̄̄	ᵒT̄r	⁺T̄a₊	⁺S̄	D̄a₊	ᵒS̄a	D̄	ᵒT̄a	⁺T̄r₊	⁺S̄	D̄a₊
m	ᵒT̄	ᵒS̄a₀	⁺S̄r	ᵒT̄r₀	⁺T̄a	ᵒS̄	D̄a	ᵒS̄a₀	D̄°	ᵒT̄a₀	⁺T̄r	ᵒS̄	D̄a

Figure 6. Table of relationships between the key of C major and all the other regions, measured in the most direct way possible.

In order to notate certain special substitutes to the dominant function, there is the sign ψ , which serves to indicate the special relationship of the antipode substitute. In the present theoretical proposal, the antipode substitute of a structure is its transposition at the tritone level, in any enharmonic spelling. This sign gives full consequence to the rationalization and mapping of the functionality generated by the voice-leading relationships found in the augmented-sixth chords and the like (see fig. 5c), and it is to be placed at the inferior right of the main D of the dominant function, at the same place where a lower-case letter would stand.

Continuing with the explanation, the signs between parentheses serve to indicate the origin of a

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[7] The concept of region is of the utmost importance to the comprehension of the principle of Monotonality (see SCHOENBERG, 1969, p. 19), according to which a musical work would be structured in one and only one tonality, being this tonality understood not as a simple diatonic place but as a universe of regions hierarchically oriented according to relationships of proximity and separation around one central region which at the end gives name to that tonality (see the

harmonic-field borrowing. In Riemann, Grabner, Mäler, and all the Brazilian texts here referenced, a relationship such as an applied (or secondary) dominant is traditionally indicated by circumscribing the harmony symbol inside parentheses, i.e.: (D), this if the applied dominant is immediately followed by the target-harmony to which it refers. In other cases, the notation ←(D) is traditionally used whenever such dominant is preceded by its target, and the notation D[x] is used when the target-harmony (represented by the x) is not immediately found present (see MICKELSEN, 1977, p. 76). For the present symbology, the marking of the D[x] kind was chosen as a general and only way of indication; nonetheless, the [x] part is relocated to the position underneath the main functional letter, and the brackets are substituted with parentheses, which then start to indicate the harmonic field (or rather, the region) of origin of a harmonic borrowing. In this way, all kinds of borrowings between harmonic fields will be notated with the function symbol of the harmony on top of another symbol between parentheses denoting the region of origin of that harmony. Since the notation between parentheses always indicates a region, there is no need to add to the symbol the sign — (see fig. 5a, as well as the tonicization cases in fig. 10, ms. 19-20). This type of notation is also very useful to indicate modulatory relationships between regions (see fig. 5b and the analysis in fig. 10). In the occurrence of a long chain of applied dominants traveling across dominants through a cycle of fifths, one can use the symbol → to simplify the visualization of the notation of the regions involved. In this case, the arrow is placed inside the parentheses substituting the notation for the region of origin of the dominant, indicating that the target of the applied dominant is the next harmony.

Completing this section of explanations, the sign ≅, placed above a main functional letter, indicates that the harmony in question operates as an enharmonic equivalent of the function indicated by the symbol (see figures 8c and 10).

3.4. Signs that indicate relationships between harmonies

Other signs serve to indicate equivalences and identities between two harmonic structures or regions. The sign = (diatonic or homograph equivalence) is used between two symbols to indicate functional relationships between modulatory diatonic pivot chords, identities between regions, as well as changes of harmonic function (see figures 3d and 5c, and the analysis in fig. 10). The aforementioned sign ≅ (enharmonic equivalence) is also used between two symbols to indicate functional relationships between modulatory enharmonic pivot chords and enharmonic identities between regions (see fig. 7a).

The sign ≈ (affinity by chromatic transformation) is used to indicate relationships of

diagrams in SCHOENBERG, 1969, p. 20 and 30).

transformational affinity between the members of a family of extraordinary dominants (see figures 7a and 7b). In this theoretical proposal, two harmonic structures are related by transformational affinity if a variant of the first structure (with added ninth, with chromatized fifth, with root omission, etc.) is identical to a variant of the second, either in a diatonic (homograph) manner, or in an enharmonic one. This procedure is capable of easily transforming a dominant into another by means of a combination of chromatic and diatonic parsimonious voice-leading motions[8], which is certainly an evolution of the "law of the shortest way" preached by Bruckner (see SCHOENBERG, 1978, p. 39). This kind of relationship is specially found between the main dominant and the dominants of the relative and anti-relative substitutes of the tonic, either in a directly diatonic way or in a borrowed one. This fact leads us to believe that such chord progressions were "learned", so to speak, from the connections between dominants traditionally practiced in deceptive cadences which involved a tonicization of the surprising element, as in the sequences [V7 – V7/vi – vi], [V7 – V7/iii – iii], for example, as well as their modal variants, either direct or borrowed.

Figure 7. Examples of the use of relationship signs.

3.5. Numbers

Similarly to the functional tradition, arabic numbers are used in several positions around the main letters to indicate complementary information regarding the harmonic structures, such as the structural bass tone and the presence of structural and semi-structural dissonant intervals added to the harmony. By structural dissonances we mean the essential dissonances of the 7th and the 9th of the dominant (do note that whenever there is a 9th, a 7th is also implied), the fifths with real chromaticism (indicated followed by the corresponding chromatic sign), and the exchanged-sixth, which can substitute the fifth of a perfect triad. All these occurrences must always be present in the symbols (see fig. 8a). As for the non-structural dissonances, which we understand as being those generated by the action of non-essential tones such as passing tones, neighbor tones, suspensions, ritardations, pedal

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[8] Such types of connection between dissonant chords, made possible by the creative maintenance of basic natural paradigms of voice leading, were already rather tentatively mapped by 19th-century theorists such as Weitzmann (see "deceptive progression" in RUDD, 1992, p. 65-66) and Reicha (see "exceptional resolutions of dissonant chords" in REICHA, 1890, p. 154, and REICHA, 1830, p. 2-5).

tones, etc., these should never be added to the symbols. In this present proposal for an analytical methodology, these non-structural dissonances are to be identified in the very score by crossing their noteheads with a solidus, as can be seen in figure 10. Nonetheless, a note usually considered as non-essential can sometimes acquire a bigger importance, almost gaining the status of a structural tone. In these cases, we consider the dissonance as being of the semi-structural type, and we add a marking of that fact to the symbol in recognition of its importance, sometimes of a historical order. Included here are the suspension sevenths (specially those on the harmony of ii), the 11th and 13th of the dominant (which when present infer at least the imaginary presence of the structural 7th and 9th), the added-sixth chord (specially that on the harmony of iv or IV, which forms a six-five chord on ii), the appoggiatura six-four chord (specially the cadential type on V) and its five-three resolution (see fig. 8b).

Thus, when placed at the right of the main letter of the structure, vertically from top to bottom in decreasing order, the numbers serve to indicate the intervals of the structural or semi-structural dissonances of the harmony, measured upwards from the root of the structure. Here the notation is practiced in a similar way as the ones by Riemann and by the majority of his revisers. Also as seen earlier, if the intervallic numbers are placed to the right of the sign ↓, it means that the intervals are to be measured downwards from the fifth of the structure (ver fig. 8c and fig. 10).

When placed underneath the main letter and positioned slightly to the right, the numbers serve to indicate the note used in the bass, with the number 1 standing for the root of the structure, 3 for its third, 5 for its fifth, etc., just in the same way practiced by Riemann and his revisers (see several instances of these markings in fig. 8). Since the basic idea behind the symbols is to analytically denote the functionality of a harmony according to its context, the information about the inversion of a harmony is generally of little interest and thus it can certainly be omitted from the symbol[9]. Nonetheless, it is sometimes useful to mark down the inversions on the symbols in cases when the inversion bears some special meaning which one wants to emphasize in the analysis, and also whenever pedagogical goals demand such precision of information, as in the case of the preparation of harmonic-writing pedagogical exercises in which one needs to specify a particular bass line. Aiming at a certain simplification, whenever a dissonant interval to be marked is occurring in the bass, it is possible to relocate it from its usual place putting it at the spot reserved for bass-tone notations (ver fig. 3f).

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[9] Anyway, I remark here that the identification of the inversional position of a harmony can be easily read from the very score.

Figure 8. Examples of the symbological use of numbers and other signs.

4. Final considerations

Currently in the process of formalization in a treatise of larger scope and breadth still under preparation, the analytical symbology and the theoretical concepts here presented have demonstrated considerable pedagogical usefulness in the explanation of the internal logic of the tonal system, both in its classical and extended versions, in the context of undergraduate and graduate courses of harmony and music analysis. As such, this symbology has always been practiced in a comparative fashion and counterposed to the classic tools of the scale-degree theory and the traditional functional harmony. To illustrate this, figure 9 shows a comparison table[10] between the symbologies of the traditional scale-degree theory and the functional theory, both in the reformed symbology version used in this research and compared to the historical functional symbols by Riemann and Wilhelm Mäler.

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[10] The references used for this table were WEBER, 1851, GAULDIN, 2004 (scale-degree notation); RIEMANN, 1903, REHDING, 2003, and MICKELSEN, 1977 (Riemann's functional notation); LA MOTTE, 1998 (Mäler's notation).

Traditional Scale-Degree Notation	Functional Notation		
	Bittencourt	Riemann	Mäler
I	$^+$T	T	T
ii	$^+$Sr	S$_p$	Sp
II = V/V	ⅅ	ⅅ	ⅅ
iii/I	$^+$T$_a$=D$_r$	Ŧ =D$_p$	T$_g$ =Dp
III/I	$^+$T$_{a+}$	Ŧ^v	TG
IV	$^+$S	S	S
V	D=°D=$^+$D	D	D
vi/I	$^+$Tr	T$_p$	Tp
VI/I	$^+$Tr$_+$	T$_p$^{v}	TP
vii^{ø7}	$^+$Ⅾ^9	Ⅾ^9	Ⅾ$^9_7$
vii	$^+$D$_a$	Ⅾ	Dg
♭VII	D$_r^o$=$^+$§	°D$_p$=§	dP=§

Traditional Scale-Degree Notation	Functional Notation		
	Bittencourt	Riemann	Mäler
i	°T	°T	t
♭II	°S$_a$	Ŝ	sG
ii°	°S^6	°S^6	s^6
III/i	°Tr	°T$_p$	tR
iii/i	°Tr$_o$	°T$_p$^{v}	tr
iv	°S =Ⅾ	°S	s
v	Ⅾ°=Ꙅ	°D	d
VI/i	°T$_a$	Ŧ	tG
vi/i	°T$_{ao}$	Ŧ^v	tg
vii°⁷	°Ⅾ^9	Ⅾ$^{9>}$	Ⅾ$^{9>}_7$
VII	D$_{a+}$	Ⅾ^v	DG
♭vii	D$_{ro}^o$=°Ꙅ	°D$_p$^{v}=^{o}Ꙅ	dp=ꙅ

Figure 9. Comparison table between the analytical symbologies of the traditional scale-degree theory and the functional theory, in three different versions (Bittencourt's, Riemann's, and Mäler's).

This pedagogical work has served as a testing ground for the development of the present proposal of symbology, and it has been always showing – or even forcing – new paths to follow, exposing faults and nearsightednesses to be corrected, this as we verify the efficiency (or not) of the structural dissecting action of these proposed tools and theoretical concepts on the works from the 19th-century repertoire. It is therefore natural that these propositions be subsequently modified and adjusted in a near future. As an exemplification of an analysis of repertoire conducted with the symbology described in this article, figure 10 shows the harmonic analysis of a passage from César Franck's Choral nº 1 in E major (1890). This analysis was prepared according to the proposal for an analytical methodology described in BITTENCOURT 2013, and it was typeset with a specific font created specially for this research.

Figure 10. Example of the harmonic analytical notation of a work from the late 19th century.

César Franck (1822-1890), Choral nº 1 in E major (1890), measures 16 to 30.

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