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MODELING AND PROJECTS: HEALTHY EATING, ENVIRONMENTAL AWARENESS AND PROPOSING A THEME

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Abstract: This work presents a proposal of modeling activities designed and implemented in a seventh-level course (12 years old). The activities include three phases: understanding modeling, modeling itself, and formulating modeling projects. These phases are intended to facilitate the modeling process. The activities are contextualized in themes of interest: healthy eating, awareness of the garbage produced, and the third phase, student proposals of a theme and problem. These proposals emphasize the development of contextualized problems where the mathematics emerge naturally from the themes. The aim of the study was to understand the difficulties and potentialities of implementing the modeling activities and was approached through a qualitative research, collecting data from the students' productions, and an interview with the course teacher. Grounded theory was used to analyze the data, from which categories emerged relating to students' and teachers' difficulties and potentialities. Results indicate that, on one hand, the proposal allowed students to visualize the mathematical difficulties and thus, support them; on the other hand, working with contextualized situations allowed the students to participate in the activities and develop communication and complementary skills. These activities allowed students to relate mathematics to real-life situations, visualizing its importance and identifying how it can be used in diverse contexts.
Keyword: Mathematical modeling. Modeling projects. Seventh level.

MODELAGEM E PROJETOS: ALIMENTAÇÃO SAUDÁVEL, CONSCIÊNCIA AMBIENTAL E PROPOSTA DE UM TEMA

Resumo: Este trabalho apresenta uma proposta de atividades de modelagem elaboradas e implementadas em um curso de sétimo nível (12 anos). As atividades incluem três fases: compreensão da modelagem, modelagem e formulação de projetos de modelagem. Essas fases têm como objetivo facilitar o processo de modelagem. As atividades são contextualizadas em temas de interesse: alimentação saudável, conscientização sobre o lixo produzido e, na terceira fase, propostas dos alunos sobre um tema e problema. Essas propostas enfatizam o desenvolvimento de problemas contextualizados em que a matemática emerge naturalmente dos temas. O objetivo do estudo foi compreender as dificuldades e potencialidades de implementação das atividades de modelagem e foi abordado por meio de uma pesquisa qualitativa, com coleta de dados nas produções dos alunos e entrevista com a professora do curso. Para a análise dos dados, utilizou-se a Grounded Theory, de onde emergiram categorias relativas às dificuldades e potencialidades de alunos e professores. Os resultados indicam que, por um lado, a proposta permitiu aos alunos visualizar as dificuldades matemáticas e, assim, apoiá-las; por outro lado, trabalhar com situações contextualizadas permitiu aos alunos participarem nas atividades e desenvolverem competências comunicativas e complementares. Essas atividades permitiram aos alunos relacionar a matemática com situações da vida real, visualizando sua importância e identificando como ela pode ser usada em diversos contextos.

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Palavras-chave: Modelagem matemática. Projetos de modelagem. Sétimo nível.

Introduction

Modeling denotes all types the activities in which relationships between mathematics and the real world are developed (BLUM; NISS, 1991). The importance of integrating modeling into mathematics learning is that the intersections between mathematics and reality allow students to make sense of mathematical content, thus facilitating and supporting mathematics learning. Nowadays, students are required to develop modeling capabilities from the first level of schooling (MINEDUC, 2015; NCTM, 2000). The current Chilean Curricular Bases (MINEDUC, 2015) give importance to developing a student's ability to model.

> The objective of developing this skill is to have the student construct a simplified and abstract version of a system that operates in reality, which captures the key patterns and expresses them through mathematical symbols. Likewise, communication and argumentative skills are central in this scenario. These are related to the ability to express ideas clearly and are is very important to understanding the reasoning behind each problem solved or concept understood (MINEDUC, 2015, p. 95).

Modeling is seen as a process in which mathematical models are selected, modified, constructed, and applied with the aim of identifying particular patterns of phenomena, objects, or situations. These can then be extrapolated to other realities (MINEDUC, 2015) with the aim that students achieve a varied representation of data in order to establish one or several methods to select and then present them through a natural language, but without losing the technical aspect of the level they are studying. In addition, if the use of analogies is considered, there is an opportunity for students to relate their experiences to formal knowledge, developing understanding, memorization, explanation of concepts, and finally, a perception of mathematical expressions that may be complex. However, as Aravena and Caamaño (2007) point out, in the Chilean education system, there are difficulties in articulating mathematics with other areas of knowledge or with students' everyday situations. Yet this connection can be made by using mathematical modeling.

A pedagogical approach that offers opportunities to develop an interactive environment is one in which students develop projects with interdisciplinary topics. Borba and Villarreal (2005) proposed the development of modeling projects in which small groups of students choose a topic of interest to investigate and, from that topic, propose a problem or question. This perspective offers the opportunity for students to make sense of the mathematics that emerges in the proposed situations by working on problems in a context of their interest, perhaps even authentic ones. In addition, students have the opportunity to get involved in a topic they would like to investigate, taking an active role in their learning process. However, there are obstacles and resistance to integrating mathematical modeling into the classroom, both from teachers and students (SILVEIRA; CALDEIRA, 2012). The teacher, on one hand, feels that he or she must leave his or her comfort zone, and students, on the other hand, must use new skills that they are not used to developing (SILVEIRA; CALDEIRA, 2012).

Some researchers have worked with modeling at different levels of education (ARAVENA; CAAMAÑO, 2007; GUERRERO; ORTIZ, 2012; VILLARREAL; ESTELEY; MINA, 2010; BORBA; LLINARES, 2012; GALLEGUILLOS; BORBA, 2018). In secondary education, Aravena and Caamaño (2007) used the modeling project strategy with students in the eleventh level of schooling (16 years old). In their work, they pointed out some initial difficulties, which were later regulated when students related mathematics to other areas of knowledge and to everyday life. Guerrero and Ortiz (2012) worked on modeling with high school students solving real-life problems. Villarreal et al. (2010) used modeling projects with secondary school students, observing the role of technologies in the development of projects. As for elementary education, we found a few studies in which modeling was developed at the seventh level of education, specifically a study in Brazil by Gavanski (1995) (in SILVEIRA; CALDEIRA, 2012) where teachers were observed guiding the modeling process with seventh grade students.

In this work we take on the challenge of integrating modeling and projects in basic education, focusing on studying how modeling activities are developed in a seventh level (12-year-old students), placing special emphasis on observing the difficulties and potentialities in the development of modeling activities. The aim of this study is to understand how the students of a seventh-level course face modeling activities, while observing difficulties and potentialities that emerge in the implementing of the modeling activities. For that, we propose a design in three phases, starting with real-life modeling problems and guiding the students to themselves propose modeling projects (BORBA; VILLARREAL, 2005).

Theoretical framework

We present the fundamental definitions of *modeling* according to Blum and Niss (1991). Modeling denotes all types of relationships between mathematics and the real world.

Regarding the concept of *problem*, a problem refers to a situation with some ambiguity, which, when asking questions, cannot be answered through the immediate use of procedures, methods, or algorithms. However, it is suggested that this definition can vary according to each individual, since what may be a problem for one student may be simply an exercise for another. The modeling process involves translating a real-life situation to mathematics, generating relationships, properties, or arguments, which is called *mathematisation* (BLUM; NISS, 1991).

Another important notion in modeling is the understanding of what a model is. A model can present its results or solutions in different forms; for example, it can be a table, graph, relation, function, equation, or simply any production having a *figural* representation (ARRIETA; DIAZ, 2015) carried out by students to explain their reasoning. In this sense, we see a model as a way of visualizing a real-life situation with a mathematical perspective (WILLIAMS; GOOS, 2013) which can be a representation of results that have some meaning for those who carry them out.

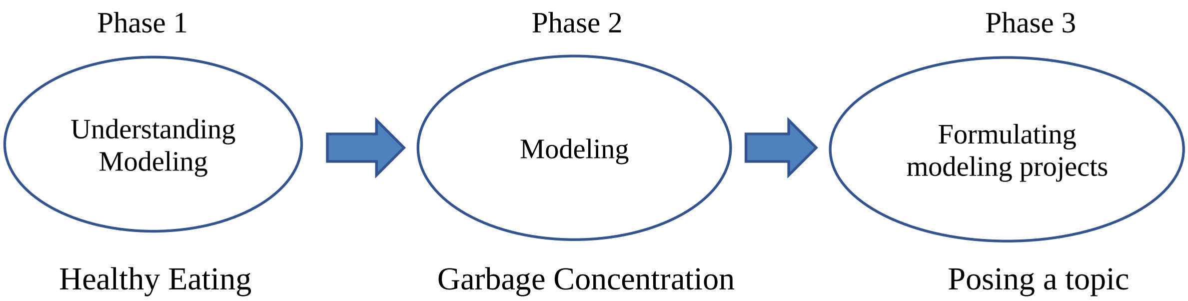
In general, there are different ways to understand modeling in the international ambit (KAISER; SRIRAMAN, 2006; WILLIAMS; GOOS, 2013). We approach a modeling projects perspective according Borba and Villarreal (2005). Modeling projects is a pedagogic strategy in which small groups of students propose a theme of interest and from it, pose a problem and solve it with the help of a teacher. This perspective allows students to mobilize their creativity and visualize the mathematics that emerge in a real context. In addition, if students investigate a topic of their interest, they will be able to engage in the modeling process. In the field of modeling projects, we use Borba's (1999) and Saviani's (1985) definition of a problem, which states that a problem has both an objective and a subjective component. The objective component refers to an obstacle faced by the group or student and the subjective component relates to the personal interest of the participants. That is, the problem may be motivated by an obstacle to be addressed or by situations of interest to the students.

However, some students face difficulties when presented with modeling problems. Researchers reported students having difficulties with modeling in relation to their understanding of the problem statement: students sometimes perform nonsensical calculations because they do not carefully read or understand the problem statement. Instead, they simply pull the numbers from the sentence and perform calculations according to some a schema that looks good (e.g. BLUM, 2012; SCHÖENFELD, 1991). Problems with group work have also been reported while developing modeling projects, since some groups delegate the project's mathematisation to the student with the most math skills (ROCHA; ARAUJO, 2012).—Such difficulties can be accentuated when underperforming and vulnerable students face modeling problems. Nevertheless, students are required to develop modeling abilities, which calls for them to complete the modeling process. Thus, in this work, we propose activities based in contextualized topics which are of current interest to students.

Methods

We propose modeling activities in three phases, from contextualized problems to projects (Figure 1). In the first two phases we propose problems around two current topics and in the third, the students themselves propose a theme and from it, they pose a problem.

Figure 1: Phases of modeling strategy



Fonte: Authors

Understanding modeling: The objective of this phase is to introduce to students to modeling problems that are attractive and challenging in a context that is interesting for them. In particular, the topic addressed in the activities relates to "Healthy Eating." This corresponds to a first approach of presenting the mathematical modeling process. Students are expected to use their problem solving, modeling, arguing, and communicating skills.

Modeling: The aim in this phase is for students to collect data to develop an authentic situation in their school, in particular here, the "Garbage Concentration." After solving the problem, we hope that students reflect on caring for the environment. The skills they use are problem solving, modeling, arguing, and communicating.

Formulating modeling projects: The objective of this activity is for student groups to choose a topic of interest, propose a problem or question, and answer it (BORBA;

VILLARREAL, 2005). Students are to use their skills to create, model, argue, and communicate in constructing the problem; tutors guide the students in the modeling process.

We adopt a basic qualitative approach (MERRIAM; TISDELL, 2016) as our research design. In basic qualitative research, the researcher is interested in understanding the meaning a phenomenon has for those involved, yet the meaning should not be discovered, but rather constructed. In our work, the phenomenon is the implementation of modeling activities. The participants were 22 students of a course in the seventh level (12 years old) of a public Chilean school. Generally these students display difficulties in their learning, low motivation, and low school performance in national tests and in university entrance tests. The school is located in Quintero commune near an area of polluting industries. The activities were developed in 3 sessions of 90 minutes each. The two first authors of this title paper took the role of course tutor-teachers to implement the activities, while the course's mathematics teacher participated by supporting the development of activities in each group; in this text we will refer to him as "teacher" of the course.

Data are collected through observations, students' productions in an Answer Guide, and a teacher's interview after the final activities' implementation. We use two Answer Guides. Answer Guide 1 for phases 1 and 2 (based in Almeida et al. (2016)), has the following steps:

- **Initial situation** (situation-problem): Understanding and contextualizing the situation, through reading, research, and previous experiences.
- **Collection of relevant information** (structure): Structuring the problem, defining the problem.
- **Solving the problem** (mathematisation): Mathematics, hypothesis formulation, variables and the mathematical model of the situation.
- **Data interpretation** (interpretation): Synthesis of the mathematical process explained in general with contextualization.
- **Final situation** (conclusion): Writing a contextualized and argued answer to the problem, and, as much as possible, a presentation.

The Answer Guide 2 for the development of modeling projects is comprised of the following steps:

- **Topic of interest**: Description of the topic of interest chosen and agreed upon by the working group.

- **Question and context**: Writing the question or problem based on the chosen topic.
- **Mathematisation**: Mathematical development of the situation--that is, definition of variables and the mathematical representation of the situation.
- **Explanation**: Explanation of mathematical development in the students' own words.
- **Alternative resolution**: Description of the alternative resolution path, if any.

The activities were carried out in three sessions of 90 minutes each, as is shown in Table 1.

Table 1: Sessions and themes

<table>
<tr><th>Session</th><th>Data and Time</th><th colspan="2">Theme</th><th>Modeling Phase</th></tr>
<tr><td>1</td><td>Tuesday, November 19, 2019. From 1:55 p.m. to 3:25 p.m.</td><td>Calories and healthy eating.</td><td>Calculate minimal calories.</td><td rowspan="2">Phase 1</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">Friday, November 22, 2019. From 10:15 a.m. to 11:45 a.m.</td><td>BMI and healthy life.</td><td>Calculate the Body Mass Index (BMI).</td></tr>
<tr><td>Environmental awareness in my classroom.</td><td>Garbage production in my classroom.</td><td rowspan="2">Phase 2</td></tr>
<tr><td rowspan="2">3</td><td rowspan="2">Wednesday, November 27, 2019. From 10:15 a.m. to 11:45 a.m.</td><td>Environmental awareness in my school.</td><td>Projection of garbage in my school.</td></tr>
<tr><td colspan="2">Elaborating modeling projects</td><td>Phase 3</td></tr>
</table>

Problem 1: Calculate minimal calories:

Stan and Freddie are two Chilean friends of 23 and 28 years old respectively. They want to know the minimum number of calories they should consume during the day. For this they go to the consultation of Dr. Yoli who took the data and analyzed the results. Then the doctor said, "Stan is 10 cm taller than the average height in Chile, while Freddie is only 5 cm shorter than Stan. Freddie weighs 15% less than Stan. And finally, Stan consumes 1,888.25 daily calories.

How tall and heavy is each person?
How many calories should Freddie eat?

According to the study of the scientific journal eLifesciences.org in 2014, it was determined that the average height of adults in Chile is 1.59 meters in the case of women, while that of men corresponds to 1.71 meters.

$$Woman\ \ K = (10 \times weight) + (6{,}25 \times height) - (5 \times age) - 161$$
$$Men\ \ K = (10 \times weight) + (6{,}25 \times height) - (5 \times age) + 5$$

Problem 2: Calculate the Body Mass Index (BMI):
There is a relationship between the mass in kilograms of each human and their height in meters. It is called the Body Mass Index (BMI) and rates the condition of each individual as underweight, normal, overweight and obese.

$BMI = \frac{weight}{(height)^2}$				
Underweight	Normal	Overweight	Obese	
$BMI < 18.4$	$18.5 - 24.9$	$25 - 29.9$	$30 - 39.9$	$BMI > 40$

Would you recommend a diet adaptation that some of the two friends follow and why?

Problem 3: Garbage production in my classroom
As part of the previous action you weighed the garbage produced in your classroom at the end of the day, which was ______ grs. According to this information, carry out this activity: Get the approximate weight of the garbage produced in your classroom per square meter in one day.

Problem 4: Projection of garbage in my school
Consider that production of garbage occurs uniformly throughout the high school and, in addition, that we have as a reference the amount of garbage per square meter in your room (calculated above). So how much garbage does the high school produce daily? What's your opinion about it?

In problems 1 and 2, the students obtain the data from the statement of the problem. In problem 3, the students themselves take the measurements of their classroom, and in problem 4, the classroom garbage's weight from the previous day is used for the problem, while the school's area is estimated from Google Maps.

Data Analysis and Results

We used Grounded Theory (GLASER; STRAUSS, 1967) for data analysis, since the categories emerged inductively from the data. The data analysis first included a description of the experience given to each tutor and the analysis of the students' productions from the Answer Guide 1 and 2, applying *open*, *axial* and *selective coding* and the *constant* *comparative* strategy. To answer our research question, the *incidents* of interest were the students' difficulties and potentialities in the development of modeling activities. The students' mistakes revealed the difficulties, and when the mistake was expanded, we found opportunities.

As results emerged, categories associated to student difficulties and potentialities related with Mathematical Knowledge (MK), Communication and Complementary Skills (CS) and the Interest and Active Participation (IP). Categories associated with teachers' difficulties and potentialities related to Guide to Students in a Modeling Processes (GT) and to Work Groups (WG). These categories are found organized in Table 3. The properties related to difficulties begin with *D* (D-MK1) and the potentialities, with *P* (P-MK1).

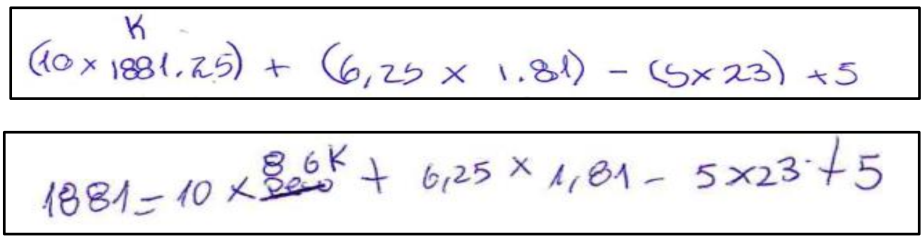
Phase 1: Understanding modeling - Healthy Eating

The application of problem 1 allowed us to identify two difficulties the students had in relation to the Mathematical Content (MK): **Posing the equation of the problem (D-MK1)** and **Clearing the linear equation (D-MK2)**. Most students did not know how to pose the equation from the problem statement. To help the students, the teachers exemplified posing and solving the equation with other values, and then the students performed it with the statement's values, thus having a potentiality of **Facing mathematical knowledge that is not necessarily seen or assimilated by students (P-MK1)**, and of **Exercising and reinforcing mathematical knowledge known to some students (P-MK2)**.

In problem 2, **Confusion of a variable with the unit of a variable (D-MK3)** emerged. The students confused the variable K (calories) with the unit of the variable Kg respect of weight (Figure 2).

Figure 2: Confusing variable with the unit of a variable.

K = (10 × weight) + (6.25 × height) - (5 × age) +5.



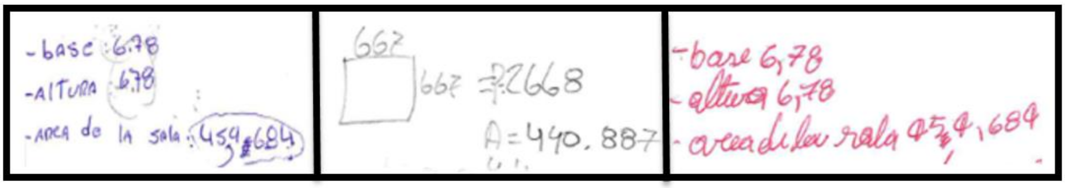
Source: Study data

In the students' answers, for the most part there was no reflection but rather information about the body mass index they obtained and the indication that Stan should go on a diet. This corresponds to **difficulties in answering questions about the problem and establishing conclusions and reflections (D-CO1)**, which also corresponds to difficulties with **Communication Skills (CO)**.

Phase 2: Modeling - Garbage Concentration

The statement in problem 3 asked students to establish a relationship between the daily production of garbage in the classroom with the area of that same room, in order to estimate the concentration of waste per square meter. The only data provided in the statement was the weight in grams of waste produced the previous day at the end of classes (420 grams). Some groups of students measured the classroom by means of measuring a floor tile with a ruler and others by using an odometer. In this way, those who used the ruler obtained the measurement in centimeters and those who used the odometer, in meters. The values obtained are quite similar to each other (Figure 3), although they did not indicate their unit of measurement.

Figure 3: Measures of the classroom



Source: Study data

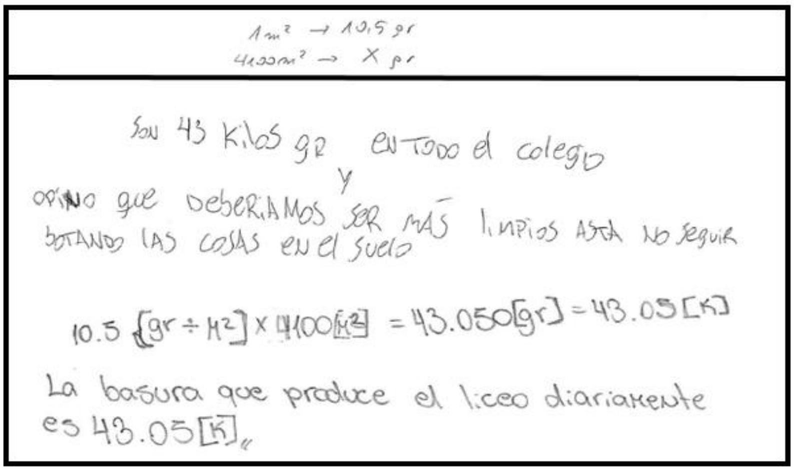
Most of the students calculated the area of the room without problems; however, when relating the data, group 2 misused the unit of mass, which was given in grams, and mistakenly used kilograms, as shown in Figure 4. This situation could be explained by the **lack of attention to the problem data (reading comprehension) D-PS1**. The focus was to calculate how much garbage was produced in one square meter, but the use of another unit of measurement in weight affected the outcome of the problem.

Figure 4: Mistake in the unit of measure

Source: Study data

The students had **difficulties in correctly formulating the relationship that allowed them to calculate the concentration of garbage (D-MK4)** (How much garbage is produced in a square meter in their classroom?). The explanations that the students groups made showed that they had divided the total area of the classroom by the total garbage weight produced on the previous day (420 gr). Instead, they were to calculate weight / area. There was no opportunity to address these difficulties (D-MK3 and D-MK4), because they were recognized in the productions after the class; thus, there was no opportunity. However, the teacher's interview showed that working with contextualized modeling problems, in which students calculated their own measurements, helped these students make sense of the mathematics used. Thus, it highlights the **importance of using units of measurement (P-MK3)** and allows students **to understand the usefulness of mathematics and its use in various contexts (P-MK4)**.

Figure 5: Calculation of garbage production



It is 43 Kg in all the school and I think we should be cleaner so that we don't keep throwing things on the floor. The daily garbage produced by the school is 43.05 kg.
Source: Study data

In problem 4, the groups were to calculate the garbage amount in their school. Figure 5 shows the results, finding 43,05 Kg of garbage per day in the school. We can see that one of the groups reflected, "we should be cleaner so that we don't keep throwing things on the floor."

As part of the statement, students were also asked to make a personal reflection about the approximate amount of garbage that their school produced in a day. Two students, reflected about reducing or reusing garbage in a way that indicated they are providing a solution to the real-life situation; three students talked about pollution, taking into account that they live in a zone with polluting industries. In general, most students seemed aware of the excessive daily waste produced by their school or were aware that they didn't need to throw garbage on the floor (for example, Figure 6).

We noticed that these reflections were more ardent than those given in phase 1, perhaps because the pollution in their community is a situation that they feel more strongly about, one they have had to live with daily. In summary, the activities contextualized in the production of garbage allowed students to make **deeper reflections on environmental awareness (P-CO1)**. Similarly, Aravena and Caamaño (2007) point out that when students relate mathematics to other areas of knowledge, they have the opportunity to develop problem interpretation, argumentation, and communication skills, which we integrate into the **Communication Skills (CO)** category.

Phase 3: Formulating Modeling Projects

The students formed four working groups and raised the following project topics: Annual hours on YouTube, Parachuting jump, Junk food, and Projection of the number of students in my school. In this work, we present two projects.

Annual hours in YouTube

Topic

This group consisted of two students. The students initially decided to find out how much time they spend on a social network application. Since they could not find that data, they decided to investigate the annual hours spent by a given number of students on YouTube. Thus, their topic of interest was "Annual hours on YouTube."

Question

The question this group of two drafted was *How many hours per year does an average person spend on YouTube?* They considered the average hours they spent per week on YouTube and asked three more classmates from other groups (for a total of 5) to consider the data.

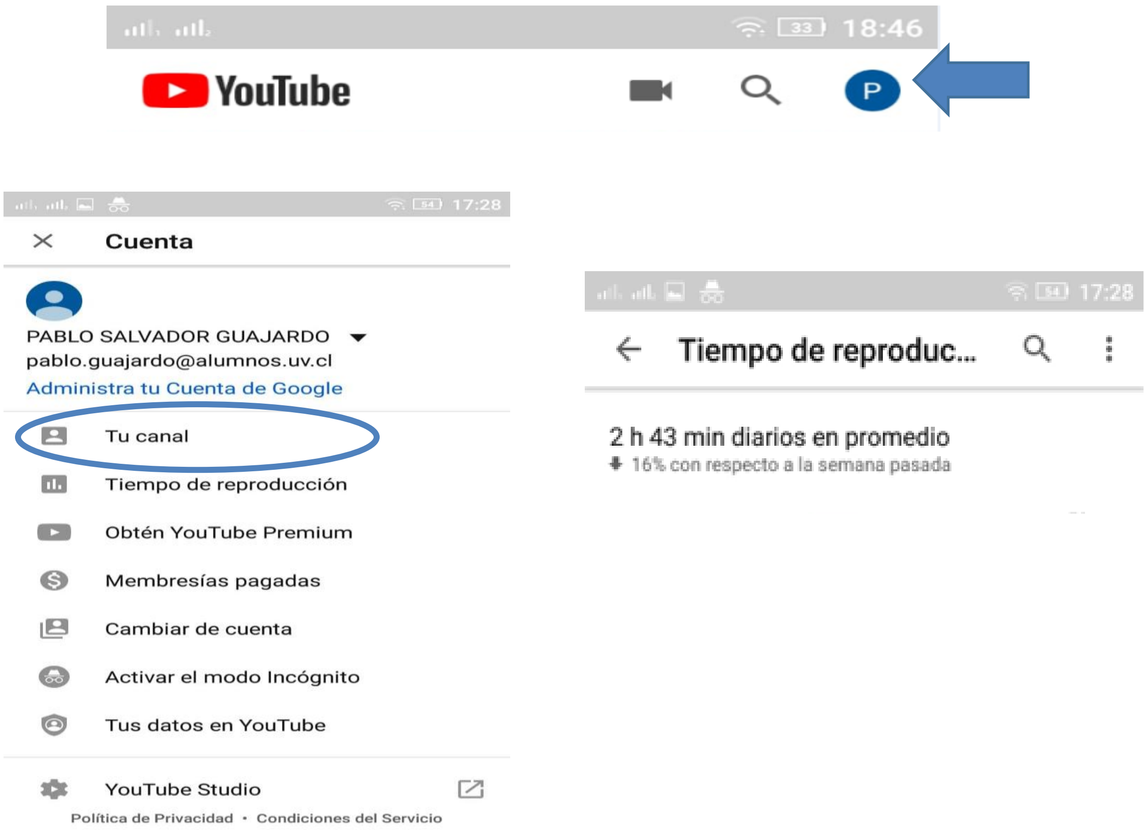
The procedure they used to find out how much time they spend on YouTube was as follows:

1. They accessed YouTube on their cellular device (with Internet access).
2. They "clicked" on their account icon.

3. Then, they chose the option "Playing time," from where they obtained the detailed information of the time spent on YouTube in the week.

The steps followed by the students are reproduced in Figure 6 for the reader's better understanding.

Figure 6: Steps to find the playback time on YouTube



Source: Study data

Mathematisation

The five students performed the process mentioned above and obtained the data of their weekly time on YouTube. With this information the group collected the data (Table 2).

Table 2: Student Average hours on YouTube (weekly)

Student	Average weekly hours on YouTube
E1	10.43
E2	3.13
E3	4.20
E4	2.31
E5	39.2

Source: Study data

The group annotated the data separated by semicolons, as follows:

10.43; 3.13; 4:20; 2.31; 39.2.

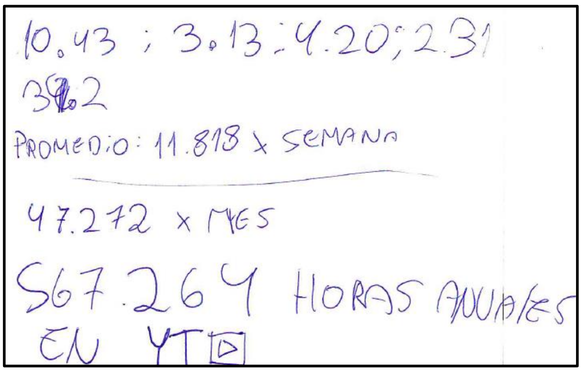
The group then proceeded to average that data (with some numerical error in the calculator input, as the last data they entered was 39.02 instead of 39.2). Next, they obtained how much time they spend on average on YouTube per month and, finally, they projected how much time they spend on YouTube per year (Figure 7). We observed that the students, in general, used the mathematics that they understand.

$$10.43 + 3.13 + 4.20 + 2.31 + 39.2 = 11.818$$

$$11.818 \ x \ 4 = 47.272 \ \ hours \ per \ month$$

$$47.272 \ x \ 13 = 567.264 \ \ hours \ per \ year \ in \ YT$$

Figure 7: Mathematisation of the group Annual hours in YouTube



Source: Study data

Explanation

The group wrote the following explanation of their procedure: "We went around consulting the YouTube playback hours of 5 people and got the weekly viewing time. Months are 4 weeks, so we multiplied it by 4 and then by 12. Which is equivalent to a year."

Alternative resolution

As an alternative resolution, they stated the following, "Google the answer," without stating further details. This group asked a simple question; however, they collected data from the YouTube app from their mobile devices. That is, they used real data and related their topic of interest to the mathematics they knew.

Parachuting Project

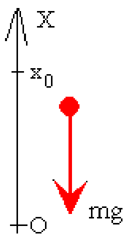
Topic and question

This group was composed of five students who were interested in studying the topic of parachuting. We noticed that, when looking out the window of the school, students could see parachutists in the distance, since the school is located in front of the Quintero airport, and it is usually possible to observe the practices of the special forces of the FACH (Chilean Air Force). Based on this, they developed the question: What is the speed at which a parachutist falls? In this instance, we observed that the group members were very interested in this topic for their project.

When it was time to mathematise the situation, the young people had doubts about what procedure to use and what to write down in the mathematisation section of the Support Guide. The tutors advised them to look for variables that can influence skydiving, such as friction, weight of the person, and gravity.

The students used the Internet to search for information on this topic and to answer the posed question, arriving at the Wikipedia page, as shown in Figure 8.

Figure 8: Information of parachuting from Wikipedia



Caída libre antes de la apertura del paracaídas

El paracaidista está sometido a la acción de su propio peso. El empuje del aire se considera despreciable ya que la densidad del aire es mucho menor que la del cuerpo. Por otra parte, consideramos que el rozamiento del paracaidista con el aire es pequeño.

Las ecuaciones del movimiento serán

$$a = -g \qquad v = -gt \qquad x = x_0 + \frac{1}{2}(-g)t^2$$

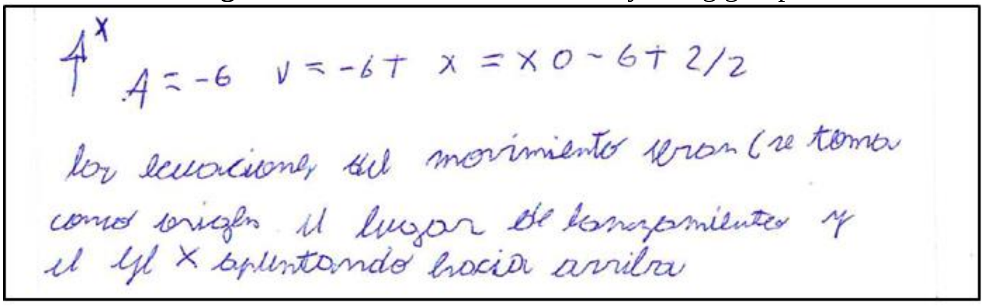
Free fall before parachute opening
The parachutist is subjected to the action of his own weight. The thrust of the air is considered negligible since the density of the air is much lower than that of the body. On the other hand, we consider that the friction of the parachute with the air is small.

Source: Wikipedia

Mathematisation

The youth wrote down the position formula and illustrated the forces acting on the skydiver's fall.

Figure 9: Mathematisation of the Skydiving group



$$a = -g \ ; \ v = -gt \ ; \quad x = x_0 + \frac{1}{2}(-g)t^2$$

The equations of motion are taken as the origin of the launch site and the x-axis pointing upwards.

Source: Data

Explanation

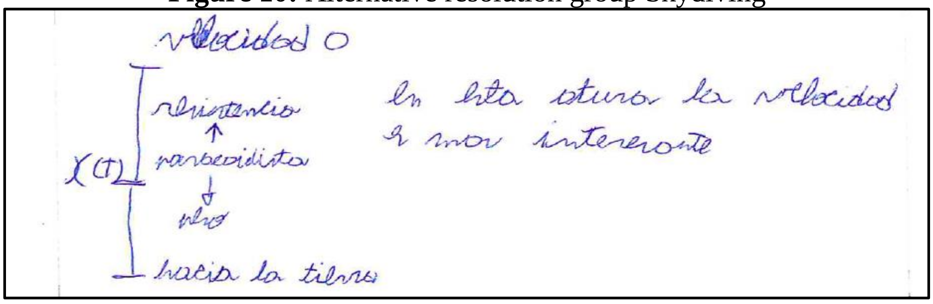
This group's explanation was: "It [speed of descent] depends on the density of the air, the height, mass, gravity, the density of the skydiver, and it depends on the suit."

The students wrote all the factors that influence a skydiver, but in the formula they showed only height, gravity, and time. At this point they should also have mentioned the equation that represents the speed of fall when the parachute opens; however, it is understandable that they left that part out, as it is complex for seventh grade students.

Alternative Resolution

As an alternative resolution, a diagram is shown in Figure 10 indicating the velocity at the moment before starting to fall whose value is 0. Then it represents the forces acting on the parachutist in his fall.

Figure 10: Alternative resolution group Skydiving



Source: Study data

We note that this group showed great interest in the proposed topic and worked hard to pose the question, search for information on the Internet, and answer the question. The mathematics that answered their question was outside their set of knowledge; however, they found the answer to that situation on the Internet and managed to explain the variables involved in that situation.

The difficulties observed in the development of modeling projects were the following: difficulties in proposing a topic to investigate (D-CS1) and difficulties in posing a problem or question (D-CS2). We have categorized these difficulties as Complementary Skills (CS), since they correspond to skills delineated by MINEDUC as complementary (complementary learning objectives). From this, the development of modeling projects offers the opportunity to develop these complementary skills (P-CS1).

In this instance we incorporated the teacher's perceptions in this study to contrast or corroborate the results obtained from the other sources. The teacher provided his perceptions by answering the following two general questions by e-mail: How do you consider that the activities were developed in the group course? and What opportunities and difficulties could you see in the realization of the three activities?

From the first question, the teacher highlighted the participation and motivation of the students in the development of the activities.

Teacher: The activities were developed efficiently and with a very motivated and participatory attitude on the part of the students, since they were focused on situations contextualized with their realities and topics of interest.

The teacher highlighted the role of the contextualization of the activities which, in his opinion, managed to promote the importance of mathematics in the students by visualizing its use in various contexts.

Teacher: It is important to mention that the key to the success of these activities was the contextualization, specifically when they [students] had to investigate with topics of their own interest and generate various proposals where the importance of science, specifically mathematics, is visualized. They were able to observe the importance of its use, and how it is used in different contexts without the need to be explicit.

The teacher mentioned that he observed the **development of efficient group work (P-CS2)**, i.e., with willingness to develop the activity and with mutual support among students.

Teacher: Real teamwork was observed by the working groups, all willing to develop the activities and support each other at all times, clarifying their own doubts and concerns.

Also, the teacher observed a greater participation and dedication of the students when they worked on projects based on the development of a topic of interest.

Teacher: Greater concentration and dedication was observed when they had to develop the topics of interest proposed by each team.

The second question asked of the teacher was about the difficulties and opportunities observed in the development of the activities. These results, together with those obtained from the other sources, are synthesized in Table 3, which summarizes the difficulties and potentialities (or opportunities) of implementing the proposed modeling activities. In parentheses we indicate the source, observing or suggesting that difficulty or potentiality.

The potentialities visualized by the teacher of the course can be summarized that first, the contextualization of the modeling situations allowed students to observe the **usefulness of mathematics and its use in various contexts (P-MK4)**. This result corresponds to one of the main goals of some modeling perspectives that focus on pragmatic issues, i.e., to visualize the usefulness of mathematics (KAISER; SRIRAMAN, 2006). We place this result in the **Mathematical Knowledge (MK)** category. Second, the groups of students worked effectively, which we integrate into the **Complementary Skills (CS)** category, and, in addition, **they participated with great interest (P-IP1)**, which we consider to be in the **Interest and Participation (IP)** category. These results are in concert with the theoretical assumptions of Humans-with-Media of Borba and Villarreal (2005) in that the learning, having the possibility of choosing a topic of interest and studying it, provided students with the potential to actively participate in modeling and, working with small groups formed by themselves, allowed them to form effective groups.

As to what the teacher observed, working in groups **allowed the teachers to attend to the diversity of opinions and students (P-WG1)**, which corresponds to the **Working Groups (WG)** category. However, the teacher also refers to the **Teacher's attrition when attending to different groups (D-WG1)**, which is also mentioned by Guerrero and Ortiz (2012), since students did not know how to work independently as indicated by their constantly requesting the teacher's help.

In summary, we found the following categories:

Mathematical Knowledge (MK): We identify two difficulties of the students in relation to the Mathematical Content (MK): Posing the equation of the problem (D-MK1) and Clearing the linear equation (D-MK2).

Communication and Complementary Skills (CCS): In this category we include student difficulties and potentialities with communication and complementary skills.

Problem Statement (PS): In phases 1 and 2, difficulties arose in understanding the problem statement or paying little attention to the data provided in the statement (D-PS1), because students used the weight of the garbage in grams without converting it to kilograms, as was indicated in the statement. However, facing modeling problems offers students the opportunity to develop and promote their understanding of the problem statement (P-PS1).

Communication Skills (CO): Some students presented difficulties in answering question about the problem and establishing conclusions and reflections (D-CO1) in the first phase. However, advancing to phase 2, the students' responses to Problem 4 (phase 2) showed that the students reflected on the garbage contamination, on the dumping of garbage on the floor at their school, and on reducing and reusing the garbage. These reflections were more earnest than those requested about healthy eating (phase 1), perhaps because they live near a contaminated industrialized area and, for this reason, identify with this theme more fervently. In summary, contextualized activities in the production of garbage allowed students to make more ardent reflections on environmental awareness (P-CO1), which we integrated into the category Communication Skills (CO) as a potentiality.

Complementary Skills (CS): The difficulties observed in the development of modeling projects (phase 3) were difficulties in proposing a topic to investigate (D-CS1) and difficulties in posing a problem or question (D-CS2). We have categorized these difficulties as Complementary Skills (CS), since they correspond to skills outlined by MINEDUC (2015) as complementary learning objectives. From this, the development of modeling projects offers the opportunity to develop these Complementary Skills (P-CS). In particular, students proposed a topic and problem (P-CS1) and they participated in groups effectively (P-CS2).

Interest and Active Participation (IP): From the observations of the development of the activities and the interview of the teacher, we observed Interest and Active Participation of

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Students in Modeling Activities (P-IP1), especially in modeling projects, which is an important potentiality, considering their previous attitudes.

Guide the students in Modeling Process (GM): A teacher-related difficulty refers to difficulties in guiding students to solve modeling problems (GM), that is, teachers excessively guiding students in solving the problem (D-GM1). This situation is called over-mediation, and it was visualized in the problem 3 when there was little time left to carry out the activity. However, over-mediation allowed us to focus on the need to improve teacher knowledge in guiding the modeling process (P-GM1).

Groups Work (GW): The teacher's interview showed that the teacher was overwhelmed due to the need to constantly monitor work groups (D-WG1), since he needed to excessively control the groups. However, group work allowed teachers to attend to the diversity of opinions and students (P-WG1), which was considered as a potentiality.

Table 3: Summary of Results

	Difficulties	**Potentialities**
Students	MATHEMATICAL KNOWLEDGE (MK).	
	(D-MK1) Difficulties in formulating the equation based on the statement. (D-MK2) Difficulties to clear the mathematics equation	(P-MK1) Facing mathematical knowledge that is not necessarily seen or assimilated by students. (P-MK2) Exercise and reinforce mathematical knowledge known to some students.
	(D-MK3) Confuse a variable with the unit of measurement of the variable or give little importance in using units of measurement.	(P-MK3) Working with contextualized modeling problems, in which students must take their own measurements, highlights the importance of using units of measurement. (P-MK4) Potential to understand the usefulness of mathematics and its use in various contexts.
	(D-MK4) Difficulty to establish the relationship that represents the concentration of garbage in a square meter $\left(\frac{weight}{area}\right)$.	

	COMMUNICATION AND COMPLEMENTARY SKILLS (CCS) Understand the Problem Statement	
	(D-PS1) Difficulties in understanding the statement of a problem or paying little attention to the data provided in the statement.	(P-PS1) Facing modeling problems offers the opportunity to develop/promote students' understanding of the problem statement.
	Communication skills (CO)	
	(D-CO1) Difficulties in answering questions about the problem and establishing conclusions and reflections.	(P-CO1) Contextualized activities allowed students to make deeper reflections on environmental awareness.
	Complementary Skills (CS)	
	(D-CS1) Difficulties in proposing a topic to investigate. (D-CS2) Students have difficulty proposing a problem (initially they proposed questions that did not make up a problem).	(P-CS1) Modeling activities promote the development of higher-level skills (proposing a topic and problem). (P-CS2) Students participated in groups effectively.
	INTEREST AND ACTIVE PARTICPATION (IP)	
		(P-IP1) Interest and active participation of students in modeling activities.
Teachers	GUIDE THE STUDENTS IN MODELING PROCESS (GM)	
	(D-GM1) Teachers experienced what we call over-mediation by excessively guiding students in the development of modeling problems.	(P-GM1) The over-mediation visualized in the development of problems allowed us to focus on the need to improve our knowledge in guiding modeling processes.
	WORK GROUP (WG)	
	(D-WG1) Teacher burnout due to the need to constantly monitor work groups.	(P-WG1) Group work allowed teachers to attend to the diversity of opinions and students.

Source: Data analysis

Discussions and Conclusions

The development of modeling processes raised difficulties with students relating to mathematical knowledge, and communication and complementary skills. These difficulties allow us to identify potentialities of the following: to tackle mathematical knowledge not assimilated by students, such as exercising and reinforcing knowledge already known to them;

to visualize the usefulness of mathematics; to develop comprehension skills of a richer mathematical statement; to develop communication skills and higher level (complementary) skills, and to actively participate with interest.

Teachers, on the one hand, experienced difficulties in guiding students in the modeling processes, which allowed them to reflect on the need to improve their knowledge to guide students in this type of activity. On the other hand, the teacher was overwhelmed by the need to monitor the groups; however, group work offered the opportunity for him to attend to the diversity of students in this course and to respond to their particular needs, which is difficult when there is no group work. In what follows, we discuss the main results of this work.

Incorporating modeling activities into this course offered the opportunity to reveal students' deficiencies in assimilating certain mathematical content. One way to address these deficiencies is by viewing modeling problems precisely and correctly.

In the first phase "Understanding modeling," difficulties arose in posing and solving an equation when students faced the first problem. This difficulty offered the opportunity for teachers to explain the contents to the course group through examples. Hence, modeling is visualized as an opportunity to address missing or unknown mathematical content, as well as the exercise and reinforcement of already known content, attending to the diversity of students in that course (BORBA; VILLARREAL, 2005).

In the second phase "Modeling," students confused the value of a variable with the unit of measurement of the variable. For example, they recorded the result in centimeters instead of meters, not making the proper conversion. The confusion of units of measurement as well as the confusion of length measurements with that of area and of that length (meter with square meter) is mentioned by Godino et al. (2002). We emphasize the need to solve fundamental education problems in which students collect their own data and take their own measurements, and then analyze the need for unit conversion. Thus, it is possible to glimpse the importance of the unit of measurement and make sense of the values and relationships that are used.

In the first two phases of the modeling problem solving, the difficulty of understanding the problem was evident. Understanding the problem's statement is also discussed by Blum (2012) where it was estimated that students work with the numbers that appear in the statement without carefully reading or understanding the problem. However, dealing with problems with a more detailed statement offers the potential to develop the ability to understand the statement of a problem.

The teachers experimented with excessively guiding students in solving problem. In this way, the opportunity for students to explore the situation more deeply was lost. This difficulty is also mentioned by Fernández-Gago et al. (2018). Furthermore, this result is related to the challenges and decision-making carried out by the novice teachers while guiding modeling projects as reported by Villarreal (2013). This difficulty is linked to the time required for modeling tasks, which is generally greater than planned, and which always seems to require the teacher to monitor the work groups.

Furthermore, students have difficulties working independently and need continuous support from the teacher. A similar situation is discussed by Guerrero and Ortiz (2012), indicating that students do not have all the needed skills to develop a mathematical modeling process independently, due to the difficulty of generating appropriate mathematical models to solve. In contrast, in Villarreal and Mina (2013), novice teachers had carried out activities prior to using them in the classroom in order to prepare students to face a mathematical modeling project.

The modeling activities designed here promoted reflection after students answered the problem. In the first phase, the students showed difficulties in establishing reflections of the results obtained and difficulty in generating conclusions or arguments. In the second phase, the reflections were more ardent. In this way, we consider that the activities promoted the development of student reflections. These reflections became more important when the students faced situations related to the garbage issue, surely because they face pollution problems daily due to living near an industrial area. Similarly, Aravena and Caamaño (2007) estimate that when students relate mathematics with other areas of knowledge, they can improve the skills of problem interpretation, mathematisation, argumentation, and communication, where they initially had presented deficiencies.

The potentialities of implementating modeling activities are framed in the role of contextualized problems and in the possibility of choosing a topic of interest to develop. These activities transformed students with low motivation into active participants who showed interest in the development of the activities. This is in agreement with the assumptions of Borba and Villarreal (2005) that when students have the possibility of developing modeling projects and choosing a topic of their interest to work with in groups, they have the opportunity to make sense of mathematics and take an active role in their own teaching-learning process.

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